The Influence of Torsional Vibrations in the Bowed Violin E-String

ROBERT WILKINS, JIE PAN, AND HONGMEI SUN

Department of Mechanical Engineering, University of Western Australia, Crawley, WA 6009, Australia
wilkinscr@optusnet.com.au, pan@uwa.edu.au, and hongmei.sun@uwa.edu.au

Abstract

This research is an empirical exploration of the torsional vibration that is the cause of the open E-string whistle. Measurements were taken directly from the violin string with a laser vibrometer. Evidence is presented that the bowed excitation of torsional vibration in the E-string is dependent on bow velocity, the influence of previously bowed A-string frequencies in the E-string, and the presence of sympathetically excited harmonics in the E-string that coincide at the torsional frequency.

INTRODUCTION

Sometimes when a player does a down-bow on a note on the A-string and then slurs across to the open E-string, the bow just slides over the string, the fundamental tone does not sound, and a high pitched whistle is heard instead. This can be very embarrassing for a performer. It can happen on any violin, even a Stradivarius. The open E-string whistle is not caused by the violin but by a high-frequency radial twisting of the bowed string.

In 1860, Helmholtz [1] showed how the bowed string vibrates laterally in a series of stick/slip saw tooth wave motions. Bow-hair contact displaces the string sideways to form a corner. When the restoring spring force becomes greater than friction, the string slips away from the bow and the corner traverses the string following the line of a parabolic envelope (see an animation of Helmholtz motion [2]). There has been extensive research on Helmholtz transverse string vibration [3, 4]. The term “lateral vibration” is used in the text, though “transverse vibration” means the same.

The high-pitched E-string whistle has been identified by Stough [5] as a torsional vibration. It differs from Helmholtz lateral vibration in that the bow has a small leverage tangential to the circumference of the string that twists the string along its length around a central axis. Torsion is an internal shear spring force that drives its own slip/stick motion against the bow. At the bowing point lateral and torsional displacements of the string occur together. Under normal playing conditions Helmholtz lateral vibration is dominant. The bow grips the string and displaces it further and for longer than it does for torsion. If and when torsion occurs alone, lateral displacement becomes only a very small accompaniment to the higher frequency torsional displacement of the string. Torsion research is challenging because torsional vibration is difficult to isolate and measure. The conditions under which torsional waves occur in the bowed violin E-string is the subject of this paper.

MEASURING BOWED STRING TORSION

Previous attempts to measure torsional waves relied on either attaching electromagnetic coils or a mirror to the string [6–9]. Adding mass anywhere on a string changes the way the string responds to excitation and distorts the measurement. This problem was avoided in the present research by using of a Polytec Scanning

Vibrometer 7.4 to focus a laser beam on the string between the bow and the bridge in the direction of bowing. Transverse string velocity is measure by a Doppler shift in the reflected laser light that returns a signal to a computer [10] where it is transformed into time history and spectral images. Except where otherwise shown, the sampling rate is 16.38 kHz for a frequency range of 6.4 kHz and a measurement time of 2 s.

Bow contact is tangential to the circumference of the string. It both twists the string in a radial motion and also pulls it laterally. This dual action means that when torsional vibration is bowed a small amount of lateral motion is of necessity phase-locked to it. The laser vibrometer records the frequency and magnitude of this motion as well as any small perturbations in the string that might signal transitional changes from Helmholtz lateral motion to torsional motion or vice versa.

The E-string whistle phenomenon presents us with an excellent opportunity to study torsional vibration in the bowed string. We have found that any violin that is fitted with a plain steel E-string is capable of making an E-string whistle. A wound string does not do this. The quality of the violin makes little difference. Two violins that were made according to bimodal plate tuning [11] were used in this research. One had a Dominant medium plain steel E-string, the other a Dominant strong plain steel E-string. In trials where the laser vibrometer was used, the violin was placed in a frame on a bench and held at the chin rest and neck to simulate the way a player would hold it. All bowing and string crossing was bowed by an experienced player using a down-bow. There is a learned knack in bowing the torsion whistle that requires using a fast light bow with rapid string crossing. Any departure from this bowing results in the E-string sounding at its normal pitch.

The whole sequence of bowing the E-string whistle is shown in Fig. 1. The measurement is taken with the laser beam on the E-string between the bow and bridge. First, the A-string is bowed at *D* = 587 Hz and then the bow crosses to the open E-string. The E-string torsional vibration starts early within the A-string decay. It appears emerging from the A-string decay and continues for ~1 s. Finally, the open E-string sounds at *E* = 659 Hz. The whole sequence is shown in a 2-s time window.

THE EFFECT OF BOW VELOCITY AND WEIGHT

According to Stough [5] the open E-string whistle is caused by a torsional vibration that results from bowing a plain steel open E-string with a

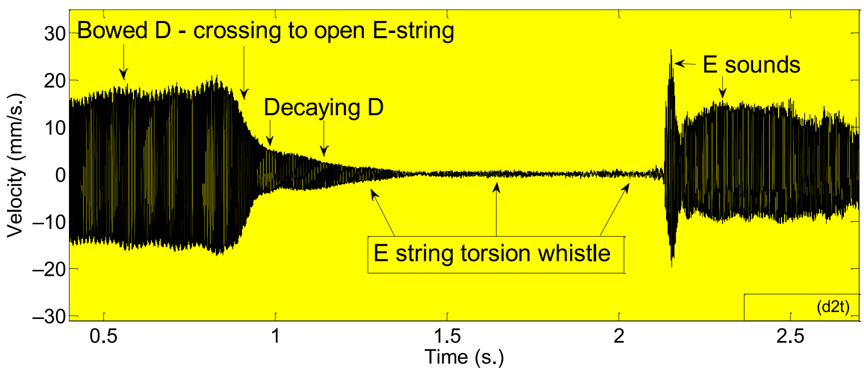
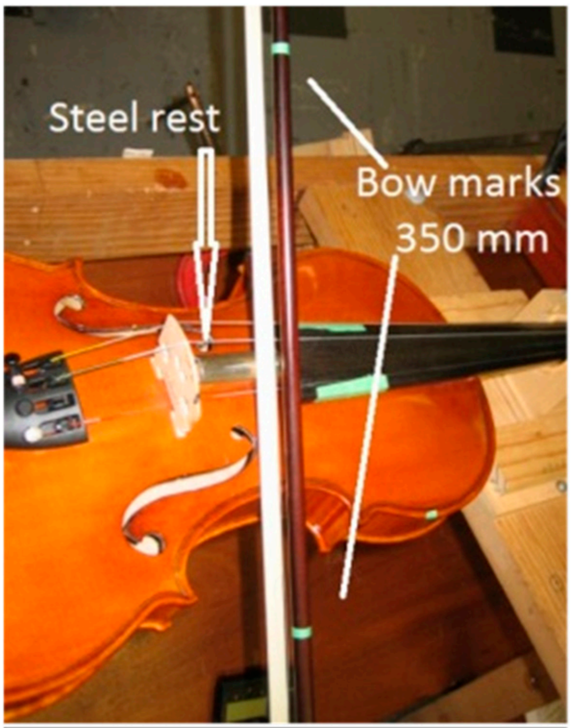


Figure 1. This time history shows the sequence of bowing the torsional whistle. Measurement is taken from the laser on the E-string between the bow and the bridge. On the left is the resonance from the bowed A-string sounding the note D = *587 Hz, which decays after the bow crosses to the E-string. The torsional vibration emerges from the decaying D and lasts for ~1 s. The torsional vibration is an audible whistle-like sound at ~4900 Hz. On the right the open E-string finally sounds at* E = *659 Hz.*

fast light bow. To verify this claim, it is necessary to locate a threshold for a combined bow velocity and force above which torsional vibration occurs and below which only Helmholtz lateral motion takes place. Bowing thresholds for Helmholtz motion have been modeled by Schelleng [12] and Guettler [13, 14]. Although these studies are instructive for bowing Helmholtz later motion, they are not readily applicable to specifying just how much bow velocity and force are required for achieving torsional vibration in a violin E-string.

The bow velocity measured in this research makes use of the player's skill in both holding and bowing the violin. For measuring bowing distance, two pieces of tape were placed on the bow stick at equal distances from the center so that they were 350 mm apart and could be seen by the player (see Photograph 1). The bow was then drawn back and forth between the two markers to keep pace with the clicks of a metronome. A down-bow was then changed to the open E-string. The metronome markings were adjusted up or down until the lowest bow velocity was found that could make the E-string whistle.



Photograph 1. Shows a stainless steel bow rest and bowing distance marks.

There was variability in bowing but over many trials this was selected out by discarding every bow stroke except one. The bow velocity threshold was defined as the one bow stroke that could achieve the torsion whistle with the least velocity.

Tests were done using a Dominant medium plain steel E-string. Three different bows weighing 57, 60, and 63 gms were used. Bow force on the string was not measured. Bow weight is something less than bow force. For example, it does not allow for the force supplied by the player. So each bow was loaded at its center with 10 gm of putty. Although this extra loading still does not give an accurate measure of bow force, it does give an additional increment of bow loading on the string that by comparison could indicate whether or not force is a relevant factor alongside bow velocity in determining the minimum requirement for bowing torsion.

Table 1 shows the bowing threshold velocity values for three bows of different weights and again when they were each loaded with 10 gm of putty.

Measurements of the minimum bow velocity for bowing a torsional vibration in the violin E-string were always considerably less for the unweighted bows than Stough's estimate of 420 mm/s [5]. The 63 gm bow was different from the other two. It had very coarse hair and a stick with less spring resistance to tightening and an opposite camber that might explain why its measured velocity differs from the other two bows. From these data it appears that there is a threshold of bow velocity relative to each bow and that additional bow weight as a component

Table 1. Torsion thresholds relative to bow velocity and weight.

Three Bow Weights (gm)	Bow Velocity (mm/s)
57	367
60	385
63	350
57 + 10	402
60 + 10	466
63 + 10	548

of force on the string leads to an increase in the velocity required to bow torsional vibration. Bowing the string below these thresholds always resulted in the open E-string sounding.

So far we have demonstrated that a combined bow velocity and weight above a certain threshold is a necessary condition for bowing torsional vibration. However, as will be shown, there are marked differences in triggering and sustaining torsional vibration when crossing from notes on the A-string. This suggests that there might be multiple factors that contribute to torsional vibration in the bowed string.

HOW RESONANCES COMBINE ON STRING CROSSING

Bowed torsion in the open E-string does not exist in isolation from other coexisting sources of vibration. The first of these vibrations comes from the A-string that is bowed before crossing to the E-string. The bowed A-string couples through the bridge and violin body and can be recorded in the E-string. It dominates the unbowed E-string but also weakly excites the E-string's natural resonance.

Figure 2 shows a forced resonance from the bowed A-string = 535 Hz with harmonic multiples in the unbowed E-string. A weak natural E-string resonance = 655.5 Hz is also shown with its harmonic multiples. It is of interest also that the 9th harmonic of E and the 11th harmonic of the forced A-string resonance add in superposition when they coincide at ~5887 Hz. This high resonance is not torsional but does illustrate the general principle of coincident superposition that later will be shown to be important for understanding how torsional vibration can sometimes interact with other frequencies.

THE FREQUENCY OF BOWED TORSION

To measure the frequency of bowed torsional vibration the violin was held in a frame and bowed above the torsion threshold. The A-string was removed to eliminate A-string influence in the open E-string. A stainless steel bow rest that had no contact with the bridge was substituted in place of the A-string (see Photograph 1). This allowed the bow to gather velocity before crossing to the E-string. The E-string was a Dominant strong plain steel string tuned to 659 Hz. The torsion whistle was heard before the E-string sounded. Figure 3 shows torsional vibration at 4970 Hz. In this case torsional vibration appears part way between the 7th and 8th harmonic of the open E-string. It therefore cannot be attributed to a natural harmonic of the E-string. In this test there is no influence from the absent

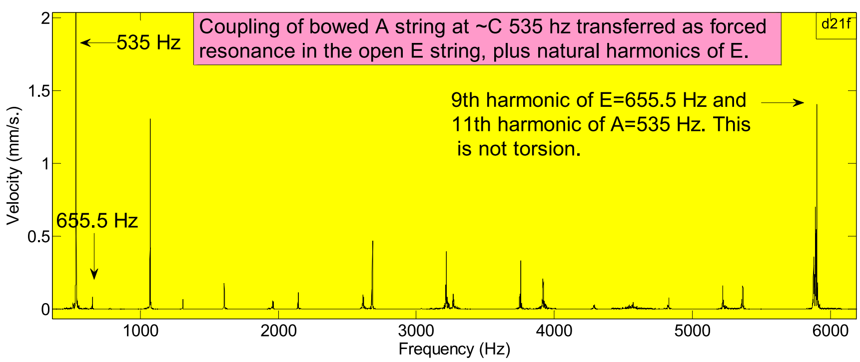


Figure 2. The laser is on the E-string near the bowing point and the A-string only is bowed at 535 Hz. The bowed A-string couples through the bridge to the open E-string. There is a forced resonance at 535 Hz with harmonic multiples in the unbowed open E-string. There are also weak natural resonances of the E-string at multiples of E = 655.5 Hz. *Both sets of harmonics combine to form a high-amplitude frequency around 5887 Hz. Note: This is not torsional vibration but does exemplify the principle of coincident superposition.*

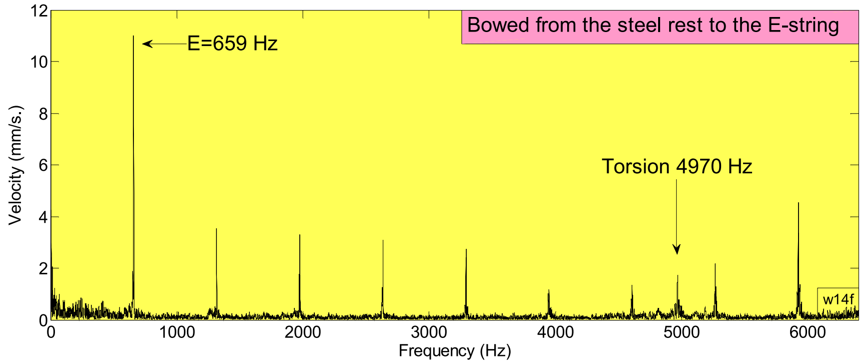


Figure 3. The spectrograph shows that there is torsional vibration between the 7th and 8th harmonic of the bowed E-string. Torsional vibration = 4970 Hz, Velocity = 1.7 mm/s. The measurement was taken from the laser on the E-string between the bridge and the bow. Torsional vibration and the E-string harmonics though seen here together are sequential events similar to the Figure 1 time history. A Dominant strong string was used in these measurements.

A-string or the violin sound box and the magnitude of the torsional vibration is very low compared with that of the open E-string.

HYPOTHESIS 1: A-STRING INFLUENCE ON TORSION

The observation from Fig. 2 that a harmonic from the bowed A-string can couple in superposition with a natural harmonic in the unbowed E-string gives rise to the most interesting part of this research. Different A-string and E-string frequency relationships were used to test whether coincident frequency couplings and/or noncoincident frequency couplings have any effect on torsional outcomes.

Our first hypothesis is that the A-string forced resonance in the unbowed E-string has a disruptive effect on the bowed start-up of the open E-string that allows the torsional stick/slip action to start first and gain dominance. When the bow is about to initiate a Helmholtz stick/slip sequence that matches the natural resonance of the E-string at 659 Hz it meets a string that is already resonating strongly at a different lateral frequency. The bow cannot get instant traction to start-up the lateral stick/slip process. If the bow velocity is high enough, torsional vibration starts first.

To give demonstrable proof of this hypothesis many tests were performed varying the frequencies of the bowed A-string so as to produce forced resonances in the E-string to see what differences they make to the start-up of torsional vibration. The laser vibrometer was focused on the E-string between the bow and the bridge to record what was actually occurring in the string before and after string crossing. The results of testing show the following two ways the A-string resonance clearly influences the outcome for torsional vibration.

First, in complete contrast to our hypothesis for the start-up of torsional vibration, if the note *E* = 659 Hz is stopped and bowed on the A-string before crossing to the open E-string, the torsion whistle never occurs, not even with a very high bow velocity. This is because the bowed A-string sets up a sympathetic resonance in the E-string with a lateral displacement that the bow stick/slip mechanism can easily continue without interruption when the bow crosses to the open E-string.

Second, when a different note is bowed on the A-string it introduces a forced resonance in the E-string. The start-up of the natural resonance in the E-string is disrupted by the forced resonance giving the torsional radial motion the advantaged to start first and set up a stick/slip rate seven times faster than required for the open E-659 Hz. If the note bowed on the A-string has a harmonic that is coincident with the torsional frequency, then the forced resonance now becomes a sympathetic resonance at the torsional frequency and the two resonances become phase-locked under a matching stick/slip rate at the bow. In this circumstance there is a greatly increased probability not only of just bowing a torsional vibration but also of increasing its magnitude and of giving it a shorter transition time at start-up. Figure 4 shows a high magnitude = 7.6 mm/s at the torsional frequency = 4883 Hz when the eleventh harmonic of the bowed A-string is coincident with the torsional frequency. Figure 5 shows two time history plots that illustrate the start-up time for torsional vibration when the bowed A-string is noncoincident and coincident with the torsional frequency.

Figure 5 shows details in two time history plots of bow string crossing from the A-string via transitions to torsional vibration in the E-string. In Fig. 5:1a and 1b, the A-string resonance is noncoincident with the torsional frequency and the transition time is ~60 ms. In Fig. 5:2a and 2b, the A-string harmonic is coincident at the 7th harmonic with the torsional frequency and the transition time is ~35 ms. The shorter transition time when a harmonic of the A-string forced resonance is coincident with the torsional frequency supports our hypothesis. We have also observed that different coincident A-string harmonic frequencies show similar shorter transition times than when they are noncoincident with the torsional frequency.

HYPOTHESIS 2: E-STRING INFLUENCE ON TORSION

The second major hypothesis extends and completes the notion that coincident frequencies influence the outcome of torsional vibration by including those that exist within the E-string itself. When the E-string is tuned up or down so that one of its natural harmonics is coincident with the A-string forced frequency and the torsional frequency, there is a dramatic increase in the magnitude at the torsional frequency. At the bowing point, the torsional stick/slip mechanism phase-locks the other two harmonic resonances to the torsional frequency. Figure 6 shows just one of many examples of this triple coincident combination. The 8th harmonic from the forced A-string = 577 Hz and the 7th harmonic of the open E-string = 655 Hz coincide with the torsional vibration at 4603 Hz in a high magnitude response = 21 mm/s. Notice that all other lower harmonics that normally belong to the two lateral string resonances are prevented from appearing when the torsional vibration and coincident harmonics combine.

Figure 6 raises the question of whether torsion controls the phase-locking of all three

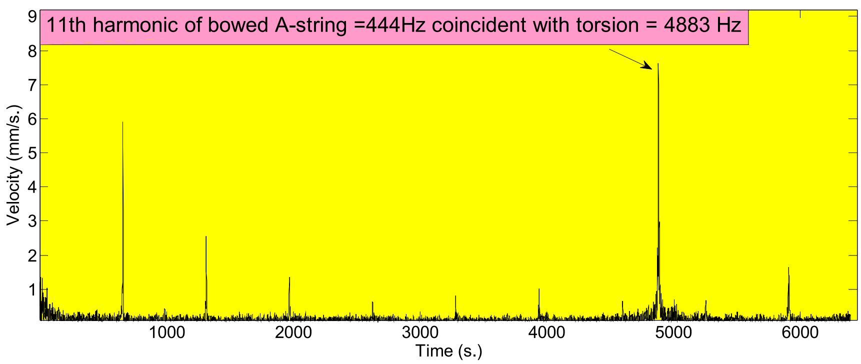


Figure 4. This spectrograph shows the E-string torsional frequency = 4883 Hz with a velocity magnitude of 7.6 mm/s. The bowed A-string was stopped at 444 Hz and has its 11th harmonic coincident with the torsional vibration = 4883 Hz. The torsional frequency falls between the 7th and 8th harmonic of the open E-string that sounds only after the torsional vibration has ceased. A Dominant medium string was used in these measurements.

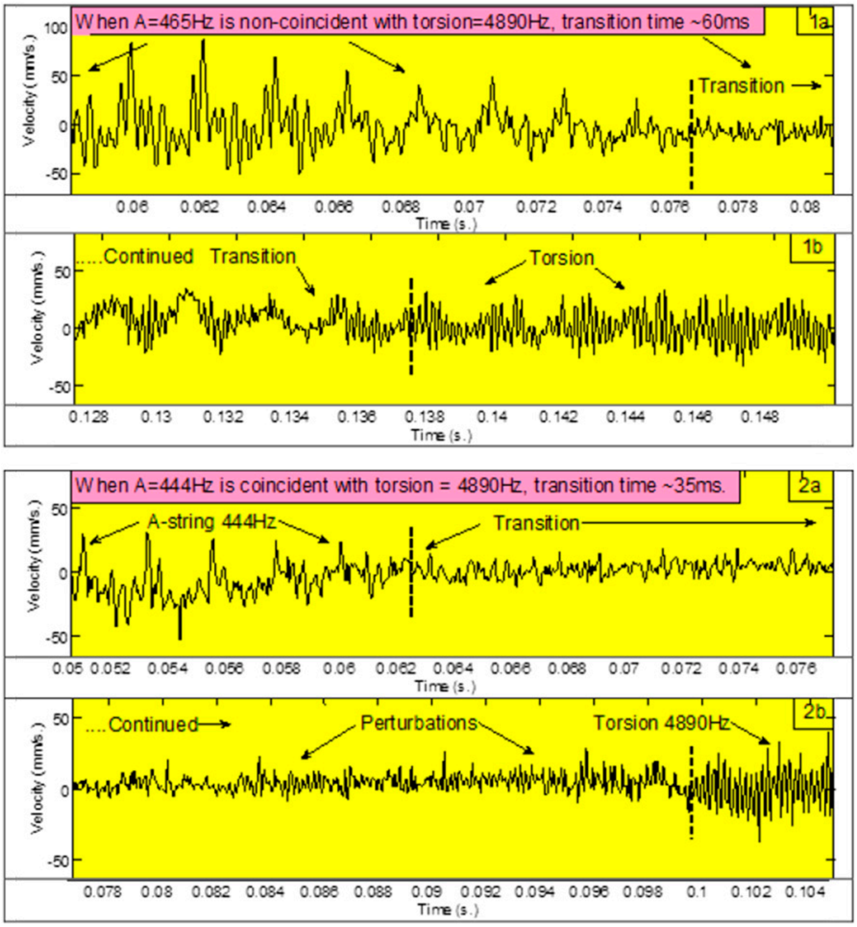


Figure 5. In sections 1a and 1b there is a transition time of ~60 ms from the bowed A-string to torsional vibration when the forced resonance A = *465 Hz is noncoincident at a harmonic with the torsional frequency. In Sections 2a and 2b, the transition time is ~35 ms when the forced resonance* A = *444 Hz is coincident at the 11th harmonic with the torsional frequency. Time plots 1a and 1b are not continuous. There is intervening time that is not shown for reasons of space. The vertical dotted lines indicate where the measurements of transition time begin and end. A Dominant strong string was used.*

coincident resonances or if it is just a simple superposition of two harmonics with the torsional vibration. This can only be answered if in a separate trial the torsional vibration can be removed while leaving the A-string and E-string resonances in place. In earlier experiments on the E-string torsion whistle, it was discovered by chance that if a small piece of masking tape 3 mm × 4 mm is either placed on or wrapped around the E-string on the bow side close to the bridge, then it becomes impossible to bow the torsional vibration no matter what bow speed is used. A series of tests were performed, with the tape "off" to include torsional vibration and with the tape "on" to exclude torsional vibration.

Figure 7 shows that when torsional vibration is excluded, by adding the small amount of masking tape to the E-string on the bow side close to the bridge, the A-string and E-string

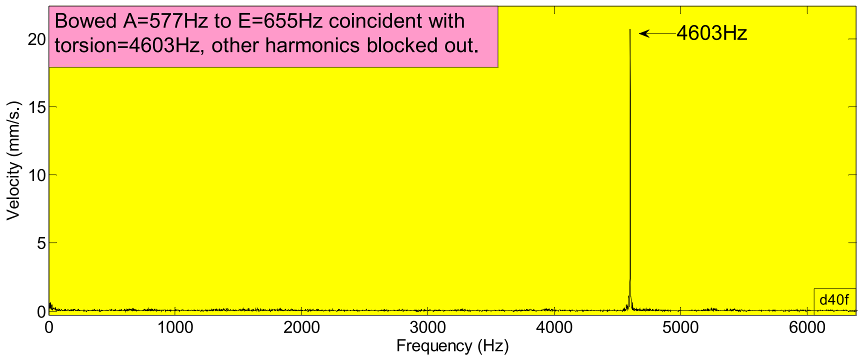


Figure 6. The 8th harmonic of the forced resonance from the A-string = 577 Hz and the 7th harmonic of the open E-string = 655 Hz are coincident with the torsional vibration = 4603 Hz. All three combine in one high magnitude resonance = 21 mm/s and all lower harmonics from the A and E strings are excluded. The bow stick/slip phase-locks all three resonances at the torsional frequency. A Dominant medium string was used in these measurements.

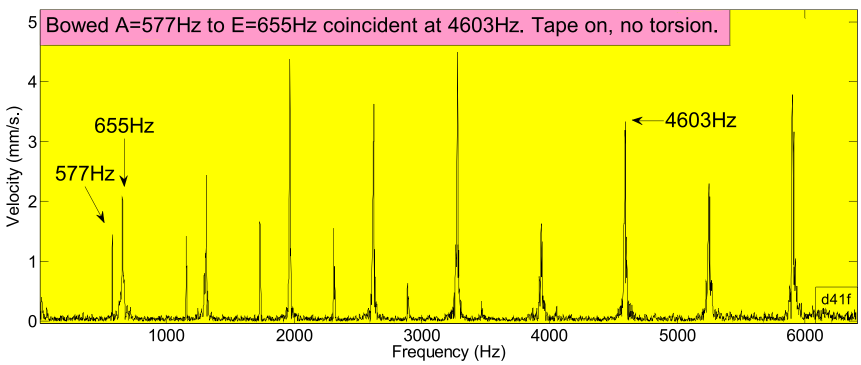


Figure 7. In this trial, following the Fig. 6 trial, torsional vibration was removed by the addition of a small piece of tape 3 mm × 4 mm on the bow side of the string close to the bridge. Without torsional vibration the A-string and E-string resonances still have a coincident harmonic resonance at 4603 Hz but the amplitude falls from 21 to 3.5 mm/s and all the lower harmonics for both strings have returned.

harmonic resonances remain coincident at 4603 Hz but their combined magnitude has fallen from 21 to 3.5 mm/s. Furthermore, all the lower harmonics of the A-string and E-string resonances have returned. It is as though the tape effectively damps torsional vibration and then in the absence of torsional vibration the Helmholtz lateral motion returns to its former harmonic spectrum.

HOW TORSION CONTROLS BOWED STRING HARMONICS

The contrast between Figs. 6 and 7 provides evidence for describing how torsional vibration holds the other coincident frequencies in a harmonic superposition. When torsional vibration is started by a fast bow and when the other string resonances have frequencies coincident with the torsional vibration some very remarkable things happen in the bowed string.

First, bowed torsion prevents Helmholtz lateral motion from getting the energy it needs from the bow. At the torsional frequency, the Helmholtz fundamental at best can only get one-seventh of its sticking time. One-seventh of bow friction is not enough to give the Helmholtz stick/slip motion the traction it needs for full lateral displacement. If the Helmholtz tuning of the string has a harmonic that is coincident with the torsional frequency, then only that harmonic can get the sticking time it needs to become excited. When the tuning of the string is noncoincident at a harmonic integer with the torsion frequency, then Helmholtz lateral motion does not get any regularly synchronized frictional sticking time at the bow and therefore cannot become excited.

Second, in violin playing, a player plays a harmonic by touching the string lightly at a node so as to damp all other resonances leaving only the chosen harmonic to sound. A node is a point on the string where the displacement of the lateral wave passes through zero. When an E-string natural harmonic resonance and an A-string sympathetic harmonic resonance combine at the torsional frequency they pass through zero at a common node. And that is just where the expected influence of torsion's radial motion around the string's central axis would be most concentrated. It appears as though torsion substitutes for the player's finger by damping at this node. As demonstrated in Figs. 6 and 7 when torsion is removed by placing a little tape on the string, the harmonic node for the coincident A-string and E-string resonances has no means to survive on its own. When torsional vibration is removed by damping the pattern reverts to the whole spectrum of Helmholtz lateral motion. We leave it to physicists to work out the complexities of how torsion might be operating in this circumstance.

Third, as a corollary of two and three above, when frequencies progressively get closer to each other and finally coincide at a common harmonic, their frequency differences have reduced to zero and the magnitude of their joint resonance has become large. This is due to the superposition principle and is seen in Fig. 6 when the A-string forced resonance and E-string natural resonance both have harmonics that coincide at the torsional frequency = 4603 Hz.

EVIDENCE FROM TIME HISTORY PLOTS

A closer look at time history plots of torsional motion will show more about the relationship between coincident lateral frequencies and torsion. First, when harmonic resonances in the E-string are coincident at the torsional frequency the stick/slip pattern becomes regular, (see Fig. 8:1). Second, when the E-string harmonic resonances are noncoincident the string's lateral stick/slip motions show perturbations. It is as though they are about to lose their connection with the bow but cannot quite do it. Then they return and spasmodically phase-lock at the repetition rate for torsional vibration, (see Fig. 8:2). Third, there are other patterns that combine motions found in 1 and 2, (see Fig. 8:3). Here there are perturbations that drift back into coincidence with the torsional frequency for ~4 ms. Such a drift could be caused by frequencies close to but not exactly coincident with torsion that superimpose briefly before parting again and/or by a small sideways shift in the bowing position relative to a node for the lateral frequency in question. There are also amplitude differences seen by the laser at the point of measurement a few millimeters away from the bow. When the resonances are exactly coincident the lateral extension is greater as in Fig. 8:1. When the resonances are noncoincident the lateral extension is less as in Fig. 8:2. Data measured from the E-string during trials of coincident and noncoincident harmonic frequencies with the torsional frequency always show similar contrasting patterns in the time history plots.

There are others variations to the three time history examples shown in Fig. 8. These variations have patterns that fall somewhere between complete coincidence and noncoincidence. An interesting one is shown in Fig. 9 because there are two alternating repetition rates for the torsional vibration. The E-string was tuned so that the torsional vibration would fall exactly half way between its 7th and 8th harmonic, which is at 7.5 multiples of E = 659 Hz. A low-amplitude torsion was expected but a much higher one resulted. In looking for an explanation, it was

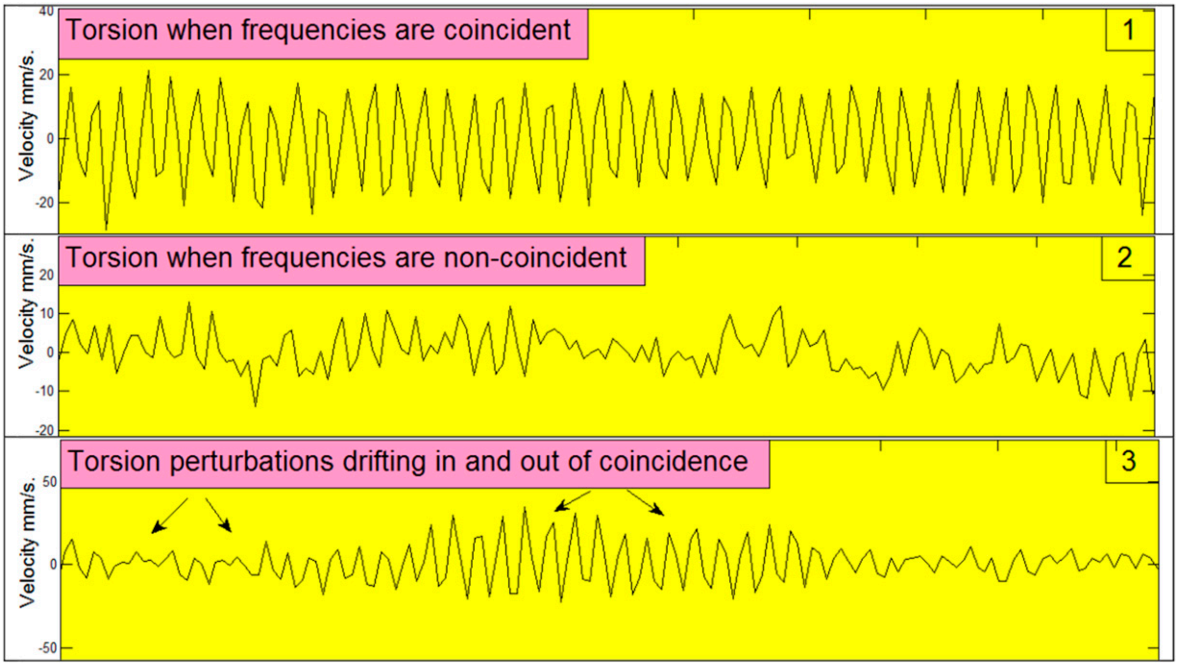


Figure 8. All three samples have the same torsional frequency = ~4890 Hz. The pattern in 1 is regular and is formed by coincident coupling of the A-string and E-string harmonic resonances at the torsional frequency. The pattern in 2 shows perturbations in the bow stick/slip continuity. The perturbations are spasmodic interruptions when the A-string and E-string harmonic resonances are non-coincident with the torsional vibration. In 2 the A-string resonance was 655 Hz, 1:7.47 noncoincident with the torsional frequency. The E-string resonance was E *= 659.5 Hz, 1:7.4 noncoincident with the torsional frequency. The pattern in 3 shows perturbations drifting in and out of coincidence with the torsional vibration for ~4 ms. A Dominant strong string was used. The time window is 0.009 s in all three plots.*

retested with the time window extended so that the second harmonic of torsion could be seen. Figure 9 (lower) holds the answer. What was noncoincident at the 1st harmonic of the torsional vibration became coincident at the 2nd because 7.5 × 2 = 15th harmonic of E and that is equal to the 2nd harmonic of the torsional vibration = 9938 Hz. This was a very rare event because tuning the E-string to that degree of accuracy is almost impossible because any error is multiplied 15 times at the 2nd harmonic of torsion. Much trial and error testing was required to replicate the exact ratio 1:7.5 so as to get exactly 1:15 for the 2nd harmonic of the torsional vibration.

In the time history plot of Fig. 9 (top), the higher alternate peaks are where the 2nd harmonic combines in superposition with the 1st harmonic. In the lower figure, the spectrograph of the 1st harmonic of torsion is noncoincident at a 7.5 multiple of E = 662.5 Hz. The 2nd harmonic of torsion is coincident at the 15th harmonic and therefore has a higher magnitude = 32 mm/s. Normally an E-string tuned as noncoincident with the torsional frequency would have a torsional magnitude of ~0.5 to ~2 mm/s but the spectral plot is up to 23 mm/s that suggests that the coincident 2nd harmonic with a magnitude of 32 mm/s changes the string dynamic to enable torsion to channel extra energy into the 1st harmonic.

By now the reader should have no doubt that coincident and noncoincident A-string and E-string harmonic resonances couple with the torsional vibration to influence the extent of its occurrence or nonoccurrence. There are other odd examples worth mentioning. It is very difficult to bow torsional vibration from the

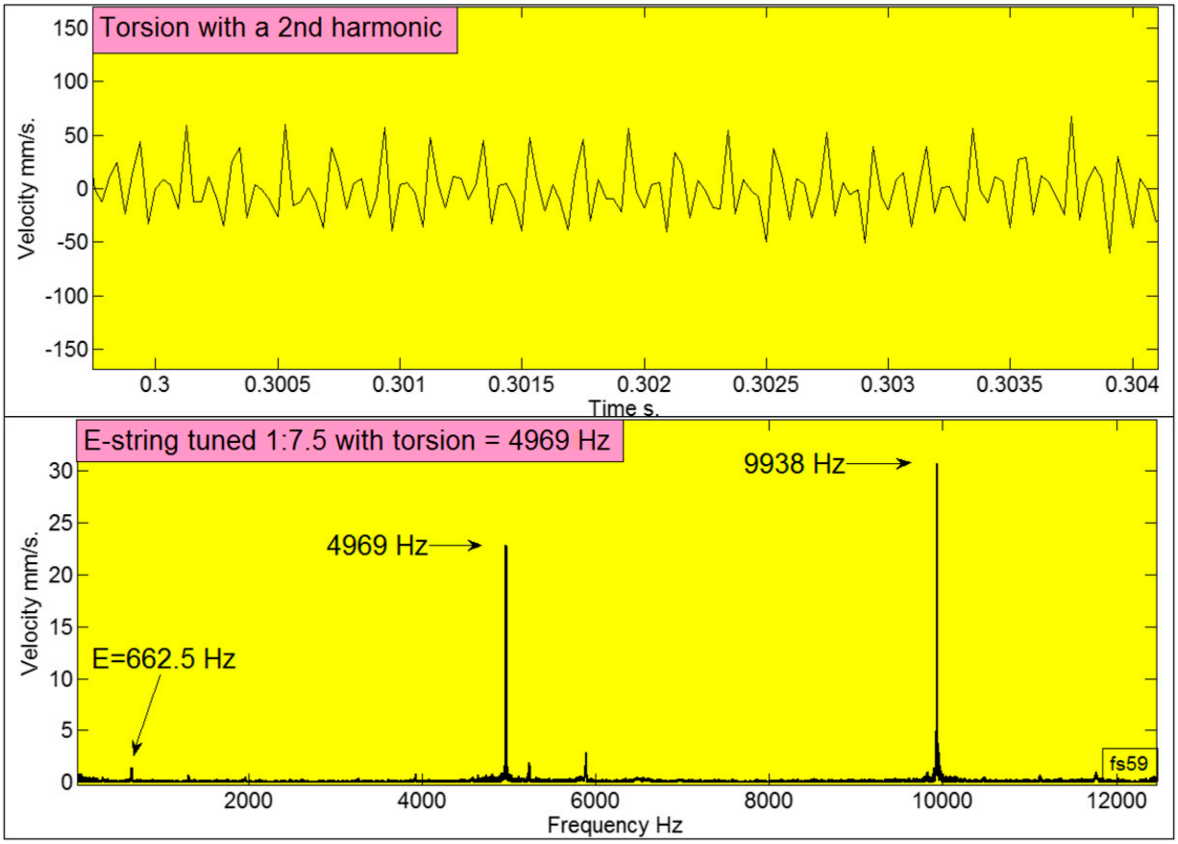


Figure 9. At the top, the time history plot shows double the repetition rate for the torsional fundamental frequency. The higher alternate peaks are where the 2nd harmonic combines in superposition with the 1st harmonic. The lower spectrograph shows the 1st harmonic of torsion = 4969 Hz as noncoincident at a 7.5 multiple of E *= 662.5 Hz. The 2nd harmonic of torsion = 9938 Hz is coincident at the 15th harmonic of E and therefore has a higher magnitude = 32 mm/s. A Dominant strong string was used. The sampling rate is 32 kHz for a frequency range of 12.5 kHz and a measurement time of 1.024 s.*

open A-string if it has a strong 3rd harmonic which it often has. This is because the 3rd harmonic of A is an E and, as was stated earlier, a sympathetic A-string resonance coupled to the E-string favors the bow connecting directly to E so that the torsional vibration cannot be bowed. At one stage during this research, with the A-string off, torsion in the E-string could still be bowed crossing from notes stopped on the D-string. Obviously, torsional vibration could not be bowed from *E* = 330 HZ, the octave below the open E-string, for the reason just stated above. However, torsion could be bowed from other positions on the D-string especially where the principle of coincident frequencies applied even when they were an octave lower than their A-string counterparts.

EVIDENCE FROM QUANTITATIVE DATA

Figure 10 is provided to give quantitative evidence in support of our hypotheses: (1) that there will be a higher response magnitude in the bowed E-string at the torsional frequency if the bow first crosses from an A-string tone with a harmonic that is coincident with the torsional frequency than if the A-string tone is noncoincident with the torsional frequency; and (2) that this response magnitude will be higher still if the

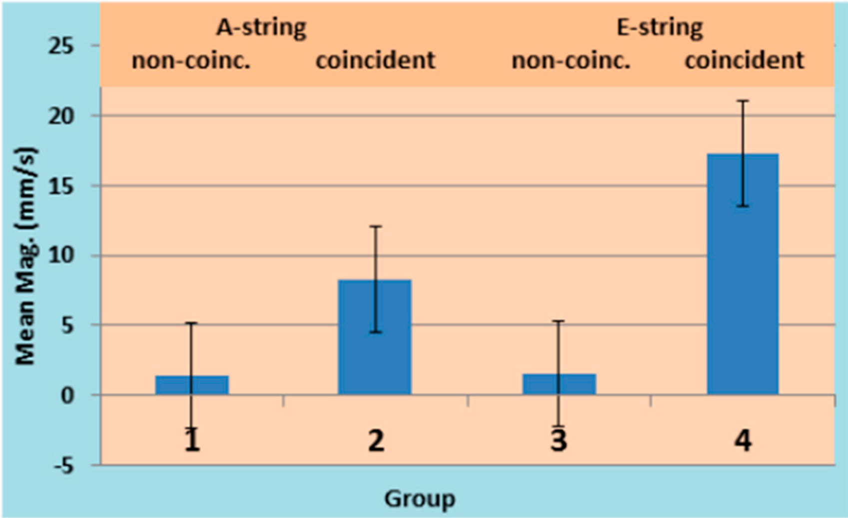


Figure 10. The histogram bars are mean magnitudes in mm/s for the E-string response to bowed torsional vibration. Bars 1 and 2 relate to hypothesis (1) and compare the E-string response when A-string harmonics are noncoincident and coincident at the torsional frequency. Bars 3 and 4 relate to hypothesis (2) and compare the E-string response when the E-string is tuned to be noncoincident and coincident with the torsional frequency. Error bars are included and a statistical test is shown below.

E-string is tuned so that a harmonic is coincident with the torsional frequency than if it is noncoincident with the torsional frequency.

Figure 10, shows histogram bars representing mean response magnitudes in mm/s for the E-string when it was bowed at the torsional frequency. Bars 1 and 2 compare this response when an A-string tone was bowed before crossing to the E-string that had a harmonic that is noncoincident and coincident with the torsional frequency. Bars 3 and 4 compare this response when an E-string harmonic is also noncoincident and coincident with the torsional frequency.

The paired comparisons 1 with 2 and 3 with 4 were analyzed using a one-tailed statistical *t*-test, with degrees of freedom = 19 and 30, respectively. Each test returned a probability <0.01. Therefore both of the hypotheses stated above can be accepted with confidence. The error bars give a visual indication of any overlap in the mean variance relating to data sets 1 and 2, and 3 and 4. The statistical tests give much greater substance to the comparisons by giving calculations of probability supporting high levels of confidence.

TORSIONAL FREQUENCY RELATED TO STRING LENGTH

Does bowed torsional frequency change with string length and tension? Torsional frequencies were measured as the string was made shorter starting with 20 mm increments. These had to be changed to 10-mm increments as the string became shorter until finally torsion could no longer be bowed at string lengths less than ~230 mm. The E-string frequency was held constant at 659 Hz and therefore had to be retuned for each reduction in string length.

Predicted torsional frequencies were calculated for string lengths between 230 and 340 mm. The formula as described in Stough's paper [5] was used for the 1st natural frequency of the torsional vibration for a string of length L with fixed ends:

$$f_1 = \frac{1}{2L}\sqrt{\frac{G}{\rho}}$$

where $G = 7.68 \times 10^{10}$ Pi and $\rho = 7{,}842\,\text{kg/m}^3$ are, respectively, the shear modulus and mass density of the string.

Both the measurements and calculations were plotted and are shown in Fig. 11. There is good general agreement between the predicted and measured torsional frequencies however the calculated frequencies are ~4% less than those measured. Unlike the Helmholtz lateral pitch of the string which changes according to length and tension, torsional frequency is inversely related to string length but did not change with changes in tension.

CONCLUSION

Torsion is an alternating radial motion that has its own stick/slip contact with the bow. Unlike normal string pitch that can be varied by changing string tension or length, torsional frequency does not change with string tension but changes inversely as the string is made shorter. This feature has allowed experimental manipulations that were used in this study to test the relationship that torsional vibration has with other bowed string resonances.

Multiple factors influence the switch from Helmholtz motion to torsional motion in the bowed steel E-string. First, there is a minimum bow velocity and force on crossing from the bowed A-string to E-string that starts the rapid stick/slip motion of the bow at the torsional frequency. This is variable according to characteristics of the bow and string.

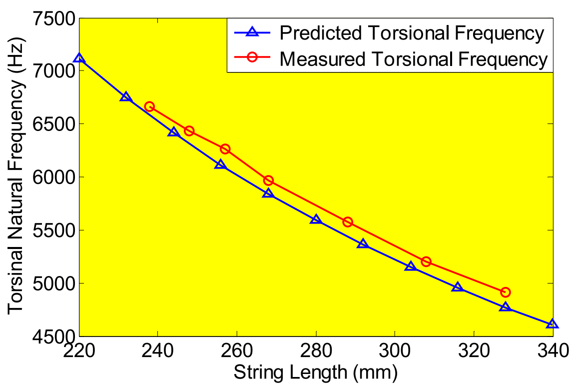


Figure 11. The (red) line shows the measured torsional frequency of a bowed violin E-string with changes to the string length while the string frequency was adjusted so as to be held constant at 659 Hz. The (blue) line shows the calculated torsional frequency for changes in string length. There is good general agreement between the predicted and measured torsional frequencies, though the calculated frequencies are ~4% less than those measured. A Dominant medium string was used for the measurements of torsional frequency.

A high bow velocity at string crossing is not always enough to guarantee torsion. At string crossing either Helmholtz motion or torsional motion can occur depending on whichever one finds a favorable preexisting resonance in the open E-string. Most often when the A-string is bowed it couples through the bridge and violin body to induce a forced resonance in the E-string. This forced resonance interrupts the Helmholtz slip/stick mechanism before it can gain traction and torsional vibration starts instead. However, if the A-string is bowed at the same frequency as the open E-string, a preexisting sympathetic harmonic resonance transfers to the E-string and that favors the start-up of Helmholtz motion. In this circumstance torsion cannot be bowed regardless of bow velocity.

Furthermore, if before string crossing the A-string is bowed at a harmonic frequency that is coincident with the torsional frequency, the transition time required to start the torsional resonance in the E-string is less and the velocity magnitude at the torsional frequency is greater. Similarly, if the E-string is also tuned so that a harmonic resonance is coincident with the torsional frequency, then the E-string velocity magnitude at the torsional frequency is very large.

The fact that bowed torsional vibration can couple with and maintain coincident harmonic resonances in opposition to the more usual Helmholtz harmonic series is because of timing. The high frequency of torsional vibration leaves insufficient time in its bowed short stick/slip window for Helmholtz lateral motion to gain traction at the bow. When bowed torsion controls the string coincident frequencies at a harmonic belonging to either the bowed A-string or originating in the bowed E-string, or when both combine in superposition, the response magnitude at the torsional frequency is greatly increased. This coupling of harmonics is held in place by torsion. If torsional vibration is removed by damping the string close to the bridge, the Helmholtz fundamental and its harmonic series comes back and the open E-string sounds.

The methodology used in this study has examined the forced and sympathetic resonances that couple at harmonic frequencies with the frequency of torsional vibration. This methodology has potential uses that could extend bowed string research to a wider range of vibrational and musical effects. These could include variations in transition response time after string crossing and tonal shifts due to the coupling and superpositioning of coincident harmonics.

Our analytical comments are those that more closely relate to what can be observed in the empirical data. There is much more theorizing that can be done and we invite others including theoretical physicists to take up that challenge.

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