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Statistical Analysis of Violin, Viola, and Cello Bridges

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Abstract

A statistical analysis of the bridges on the www.violinbridges.co.uk database was conducted. The raw data were provided from a database export and manipulated to fit into an Excel spreadsheet. The data were then split into different spreadsheets for the violin, viola, and cello. The data were plotted for consistency to ensure that outliers were removed or confirmed. In a few isolated cases, checks were made against the photographs and some corrections were made. The stamp data were also cut in and sorted with the numerical data. There are a few bridges without names. These represent some unstamped bridges and some names that were illegible. A background summary of research related to bridges was also compiled. The intent of this was to provide a context with which to review the implications of all of the data analysis.

HISTORY OF VIOLINBRIDGES.CO.UK

The idea of violinbridges.co.uk was sown by a brief conversation between Gerard KilBride and Klaus Klepper about bridge weight affecting tone. Both were studying at Newark School of Violin Making. Many years later, after fitting more than 1,000 bridges, Gerard was working with the talented restorer Mick Quinn. They revived the idea of the violinbridges.co.uk Web site. Mick had an interest in programming and Gerard loved a project, which led to the first Web site in 2004. Bridges were submitted over many years by various luthiers, and are therefore broadly screened. The bridges added were from well-known makers, highly original, or showed evidence of good quality workmanship. The bridges presented are, therefore, not statistically random samples. Since submission was elective, it is suggested they represent examples of interest and deliberate design.

MEASUREMENT POINTS

The actual measurement points are shown in Fig. 1 in numerical order.

The data are recorded with a database template. The template varies slightly from the Excel spreadsheet. A blank template is shown in Fig. 2. There is one significant terminology variation; widths and thicknesses have been interchanged. For those that have actively entered data, this might cause a little initial confusion.

DEVELOPMENT OF BRIDGES

Development of the conventional violin bridge has been reverse engineered by Akihiro Matsutani. His article, "Study on Bridge of Violin by Photoelastic Observation and Frequency Analysis" [1], shows that the modern elements, such as the kidneys and heart, have definite effects on the stress distribution in the bridge. It follows that vibrational behavior will then change and, hence, tone and sound. The work was performed with clear epoxy bridges that have a photoelastic effect. The original pictures of the bridges in Fig. 3 were difficult to interpret since the stress bands are close together and have no values. The data have been simplified somewhat, to highlight the interpreted major features. The

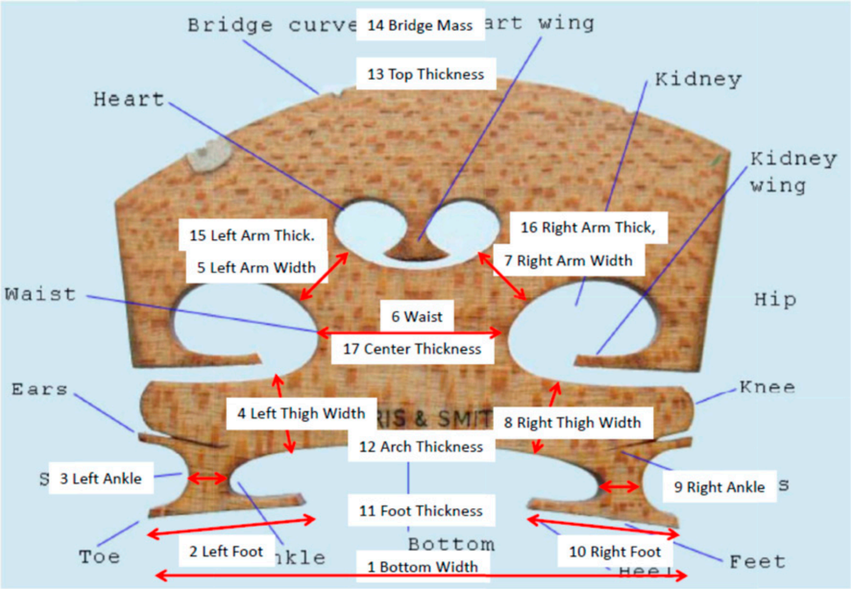
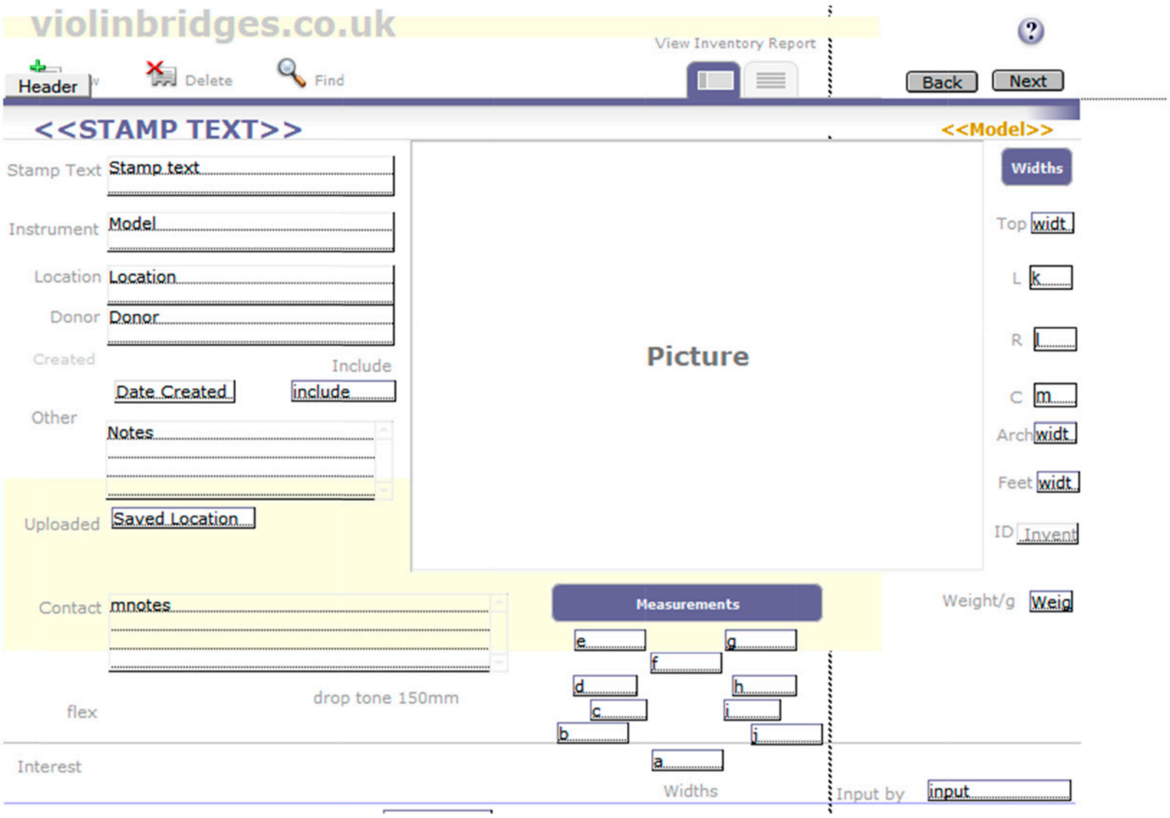


Figure 1. Bridge measurements.



Stamp Text: Stamp text

Instrument: Model

Location: Location

Donor: Donor

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Include: include

Other: Notes

Uploaded: Saved Location

Contact: mnotes

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R: l

C: m

Arch: widt

Feet: widt

ID: Invent

Weight/g: Weig

e, g, f, d, h, c, i, b, j, a

Input by: input

Figure 2. Bridge measurement input.

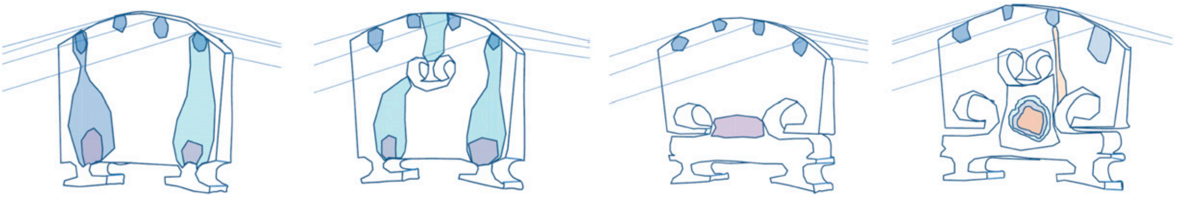


Figure 3. Photoelastic effects on bridge.

geometry of the bridges has been changed progressively. The first item to note is that there is a stress concentration under all of the strings. It also shows that without the waist, there are two downward stress concentrations onto the ankles. Without a waist, the heart bends the stress concentration to emphasize the center strings. The addition of a waist concentrates stress and allows the bridge to bend or move transversely or "dance" laterally. With transverse movement, the stress underneath the feet will change as the upper bridge moves from side to side. The heart not only lightens the upper bridge but also serves to concentrate stress in the middle. Legs will allow the bridge to "squat" up and down. Forward and backward bending is possible with all geometries.

Any area that is highly stressed is more likely to fail and will also be an area where small changes will have more pronounced effects. Conversely, if one is trying to reduce weight without affecting strength, areas that are lightly stressed can be altered.

BRIDGE BLANKS

Modern bridges are bought as precut manufactured blanks from a number of different manufacturers. They come in an array of sizes. Luthiers custom fit and trim bridges to fit each instrument. Before recent times, bridges were made from scratch. There are also some lovely modern examples of bridges cut from scratch to correct arching problems, in cases where no standard blank would work.

Vibrations and Violins and Violas

Rodgers and Masino (see References on violinbridges.co.uk) have shown that the operation of a bridge is more complicated than is readily apparent. Following Chuck Traeger's law: on a double bass (read string instruments in general), everything is vibrating. The work that Rodgers and Masino [2] did was a finite element analysis (FEM) of the vibration modes of both violin and cello bridges. The violin bridge will naturally "dance" in a number of different modes above critical (first mode) frequencies when excited. The string and the top of the violin have many harmonics in them, and the net result is that the bridge is always vibrating. Unfortunately, a printed document is not quite as easy to interpret as a video. At the time this work was carried out, video software was not readily available. They have not shown the higher order modes and there are a lot of them. Vibration engineers are typically most interested in the lower modes as they are usually the strongest. An example would be the body of a car. If the car body vibrates (rattles), it makes for an unpleasant driving and/or passenger experience. The first three modes are shown in Fig. 4.

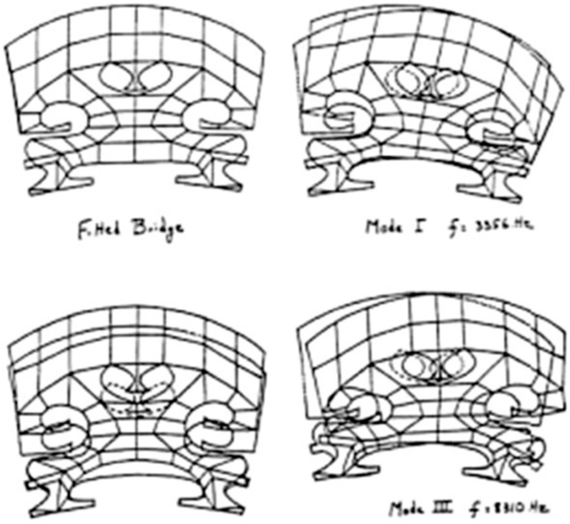


Figure 4. Violin bridge in plane vibrating modes patterns before tuning [2].

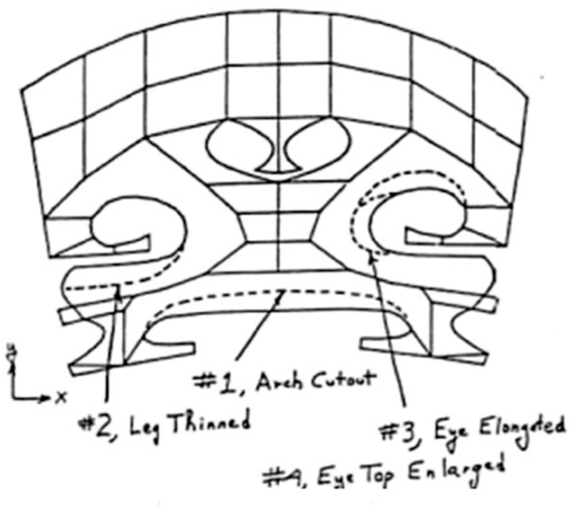
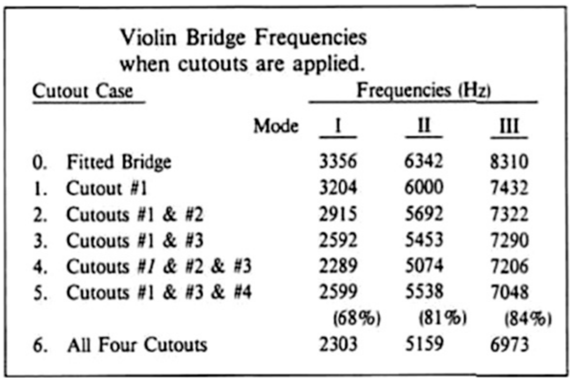


Figure 5. Finite element mesh [2].



Violin Bridge Frequencies
when cutouts are applied.

Cutout Case	Mode	I	II	III
0. Fitted Bridge		3356	6342	8310
1. Cutout #1		3204	6000	7432
2. Cutouts #1 & #2		2915	5692	7322
3. Cutouts #1 & #3		2592	5453	7290
4. Cutouts #*1* & #2 & #3		2289	5074	7206
5. Cutouts #1 & #3 & #4		2599	5538	7048
		(68%)	(81%)	(84%)
6. All Four Cutouts		2303	5159	6973

Figure 6. Bridge measurements [2].

Violinmakers have been making use of these vibration modes by trimming bridges. The normal pattern was analyzed by Rodgers and Masino based on their discussions with luthiers.

Much of the data collected in the database gives an insight into how the bridges have been cut (see Fig. 5). The "leg thinning" is represented by thigh thickness. The waist width gives an indication of how much the kidneys have been expanded. The degree of arch cutout is not directly measureable in the database.

The analysis shows that the frequency that the vibration starts at can be manipulated. In general, cutouts lower the frequency at which the various modes occur. The basic data have been extracted and presented in Fig. 6.

Rodgers and Masino's data suggest waist (–612 Hz) is the most important, followed by thighs (–285 Hz), followed by the bottom arch (–152 Hz), with the top of the kidney (+7 Hz) having almost no effect. The cutouts lower the frequency at which the bridge starts to dance, and presumably, the dance gets more intense with less mass. Once it starts to dance, the motion contains rotation, and this means that where the weight is located becomes important, i.e., the moment of inertia. This is more complicated to evaluate because distance and mass must be multiplied, and this means a more complex calculation involving integration.

Vibration and Cellos

The physical arrangement of a cello is quite different. Rodgers and Masino addressed the vibrations on cellos (Fig. 7). The first part is for French bridges.

The cutouts that are common, based on discussions with luthiers and which were tested with modeling, are shown in Fig. 8.

There is a great deal of similarity to that of the violin bridge. The waist can be thinned, the arch extended, and the thighs thinned. Although the ankles can be adjusted on the violin, this is not a major change. On the cello, significant thinning of the ankles is possible—although the ankles are really a significant portion of the leg.

As with the violin, thinning affects the start of vibrations (Fig. 9).

Rodgers and Masino's work also covered Belgian bridges and these are really a separate geometry.

In order, from the largest to the smallest, changes in Mode 1 frequency were as follows: cutout #1 –302 Hz, cutout #4 –142 Hz, cutout #3 –123 Hz, and cut-out #2 –103 Hz. All four cutouts dropped the Mode 1 frequency by –406 Hz. The author is not aware of any direct studies on the effect on tone; however, it seems reasonable to infer similar trends to what is seen on the violin (Fig. 10).

There are similar cutouts that can be made on a Belgian bridge (Fig. 11).

The various cutouts were evaluated using the FEM program as follows (Fig. 12).

Note that the Mode 1 frequency for the Belgian bridge is lower than that of the French bridge (1543 vs. 1642 Hz). Cutout #1, which

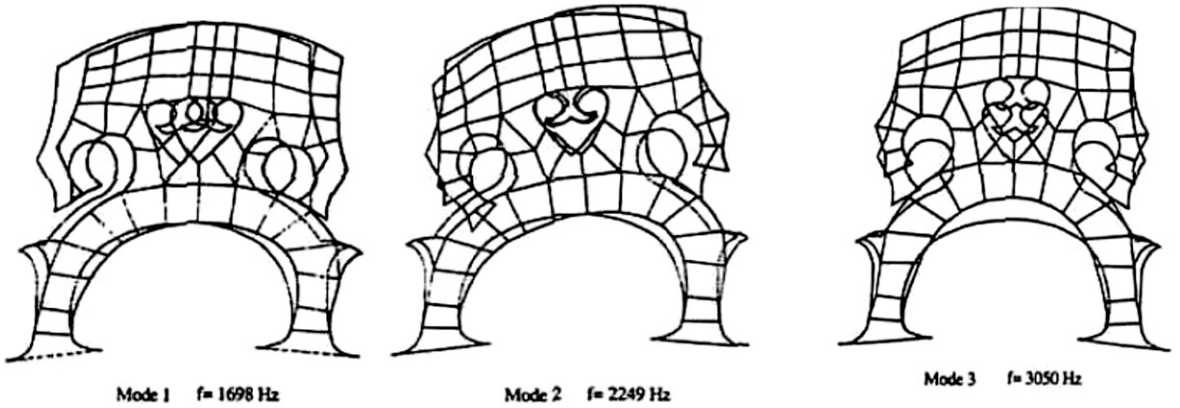


Figure 7. Bridge measurements [2].

is the legs, has the biggest effect at –302 Hz, which is slightly more than the effect on the French bridge. After cutout #1, the order of effectiveness for Belgian bridges is different from that for French bridges. The second biggest effect is cutout #2—extending the arc at –169 Hz. This is somewhat larger than the second largest change on the French bridge of –142 Hz. The third largest change was cutout #4—reduction of the thigh, at –19 Hz. This is not a large effect. The last change evaluated, cutout #3—narrowing of the waist, had almost no effect at –1 Hz. The cutouts on the French bridges all featured reduction of Mode 1 by over 100 Hz. The total changes amount to –354 Hz, which is only moderately less than the total change of –402 Hz for the French bridge via cutouts. The Belgian bridge was 99 Hz lower to start.

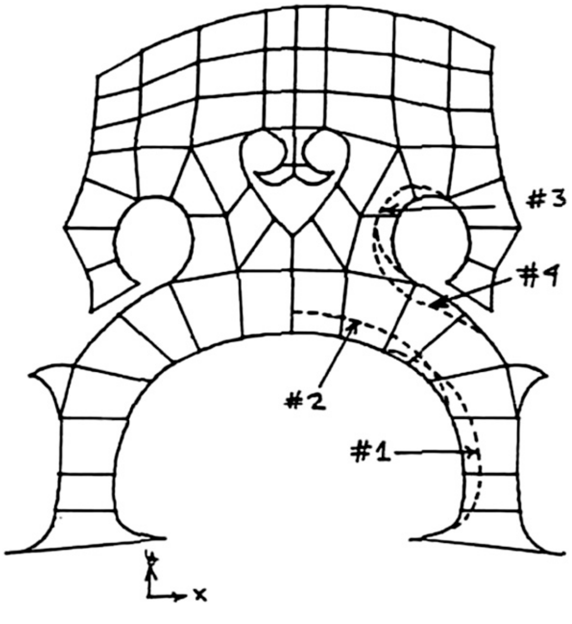
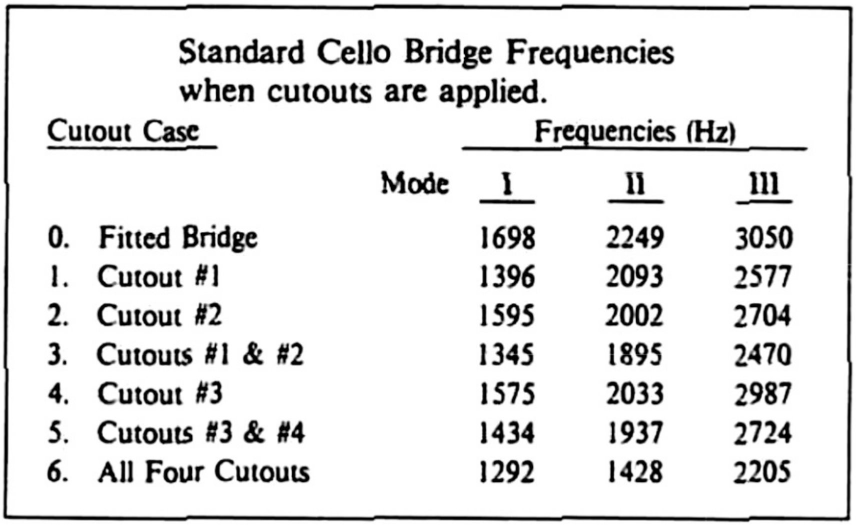


Figure 8. Finite element mesh [2].

Expected differences, attributed to Cello expert Robin Aitchison, describe the expected outcomes that the French and the Belgian model bridges can give on his Web site:

> The legs of the French bridge account for approximately half its height and within this basic design there is plenty of latitude for the luthier to choose slightly different shapes and thicknesses to control the tonal outcome. The French bridge is often a good choice for bright-sounding cellos. The reduced mass of wood above the Belgian heart produces a sound that is brighter and more open than the French bridge - and is often louder. The Belgian bridge emphasises the upper register of the cello and can also be used to make the sound of gut G and C strings more crisp and clean. Cellos with an inherently dark sound often benefit from the fitting of a Belgian bridge.

The database was accordingly split based on the bridge type. It is also clear that the geometrical differences in cello bridges do have an effect on tone. The observations of Rodgers and Masino show less options for control with the Belgian bridge; however, slightly more reduction in Mode 1 frequency can be obtained. The lowest Mode 1 for the Belgian was 1189 Hz vs. 1292 Hz for the French bridge.



Standard Cello Bridge Frequencies when cutouts are applied.

Cutout Case	Frequencies (Hz) Mode I	II	III
0. Fitted Bridge	1698	2249	3050
1. Cutout #1	1396	2093	2577
2. Cutout #2	1595	2002	2704
3. Cutouts #1 & #2	1345	1895	2470
4. Cutout #3	1575	2033	2987
5. Cutouts #3 & #4	1434	1937	2724
6. All Four Cutouts	1292	1428	2205

Figure 9. Bridge measurements [2].

Expected Effect of Cut-Outs and Weight Reduction

The removal of material is the reverse of muting. And we know a bit about this from experience and articles on mutes. The summary is that a mute generally makes the sound more dark and of lower volume. In fact, a heavy practice mute really cuts the volume a lot. The graphs in Fig. 13 are from Kenshi Kishi: "Influence of the weight of mutes on tones of a violin family" [3]. Note that multiple modes occur simultaneously and more than one "dance" occurs simultaneously with different strengths.

It follows that the reverse, reducing the mass of the bridge, increases volume and makes the violin brighter. Too much brightness and volume can make a violin sound harsh to the player, although this does not seem to affect audiences nearly as much.

The Big X

One way to simplify the description of the bridge structure is to view the load bearing of the bridge as a big X. There is an implicit assumption that there is a support at the top that extends all the way across the top to hold up the

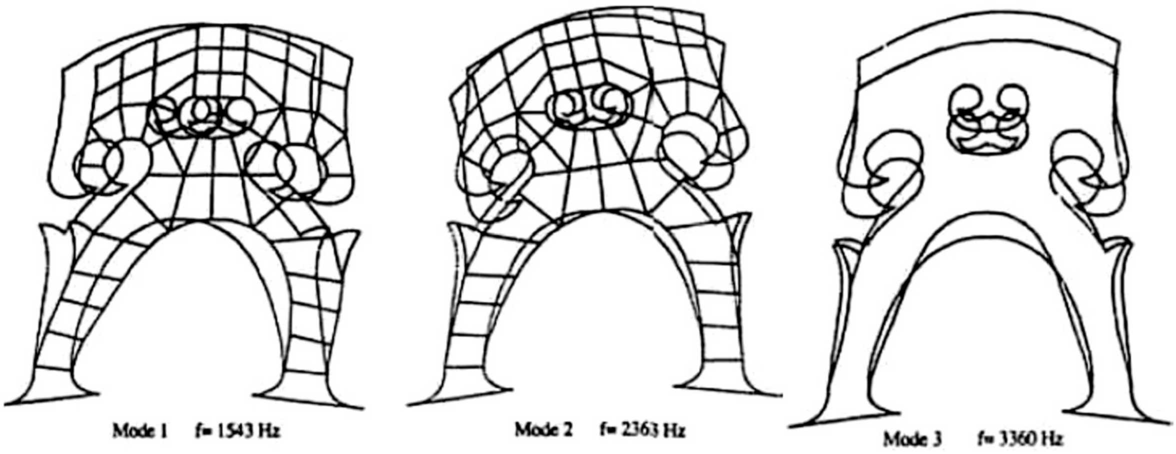


Figure 10. Bridge measurements [2].

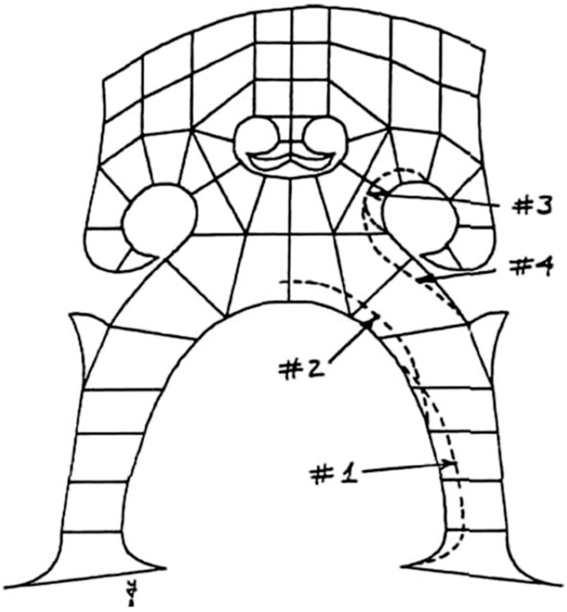


Figure 11. Bridge measurements [2].

strings. The photoelectric photos from Matsutani show there is a small stress concentration underneath each string in this support. This "big X" is shown in Fig. 14.

So making the thighs and arms thinner makes the bridge more mobile, and this changes the dance. The center of the X is the "core," and the thickness and width of the waist will have a big effect on vibration. These data are included in the database. This is consistent with the Rodgers and Masino work.

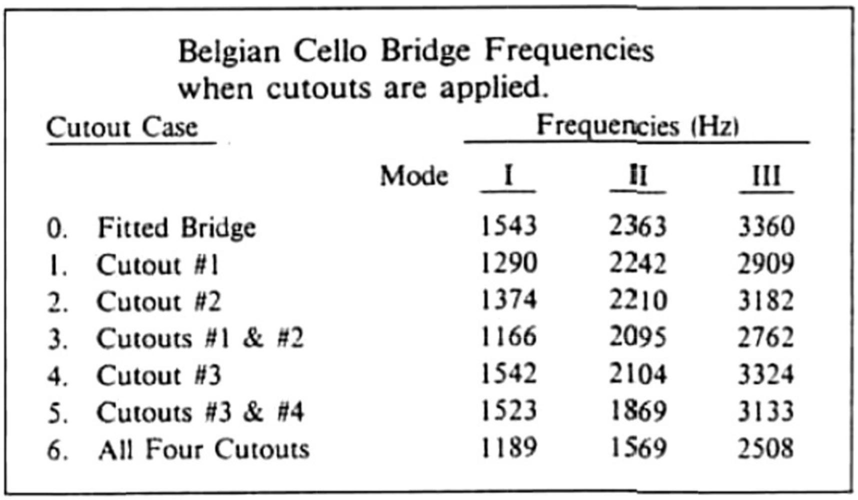
If one tries to modify the structure of the X, other cutouts are possible, and they are shown in Fig. 15, as presented by Jansson et al [4].

The essence is to pick out areas that are superfluous to the X; the purple, orange, green, blue, and red areas can be easily cut down, in addition to the arch and kidney cutout. The ankles will act as mini columns, and this suggests they will not change the dance much. The author has not seen a model on this yet. The feet and ankles do provide an area to potentially remove weight. Basically, cutout and thinning adjust the effective stiffness of the waist, and the mass that has to be moved. This changes the dance of the bridge.

Vertical and Horizontal Profile

Figure 16 has been extracted from a college text by Johnson and Courtnall [5] on violin construction. As can be seen, the motion of cutting leaves the center part thicker and the edges somewhat thinner. The vertical profile shows a curved profile with the slope of the profile becoming steeper toward the top. Some bridges have a relatively little profile and appear to be shaped more like a narrow capital A when viewed from the side, usually with a slightly squared top. In other words, the front face of the bridge is planed flat.

The suggested methodology is the use of a chisel. Clearly, this can also be performed with



Belgian Cello Bridge Frequencies
when cutouts are applied.

Cutout Case	Mode	I	II	III
		Frequencies (Hz)		
0. Fitted Bridge		1543	2363	3360
1. Cutout #1		1290	2242	2909
2. Cutout #2		1374	2210	3182
3. Cutouts #1 & #2		1166	2095	2762
4. Cutout #3		1542	2104	3324
5. Cutouts #3 & #4		1523	1869	3133
6. All Four Cutouts		1189	1569	2508

Figure 12. Bridge measurements [2].

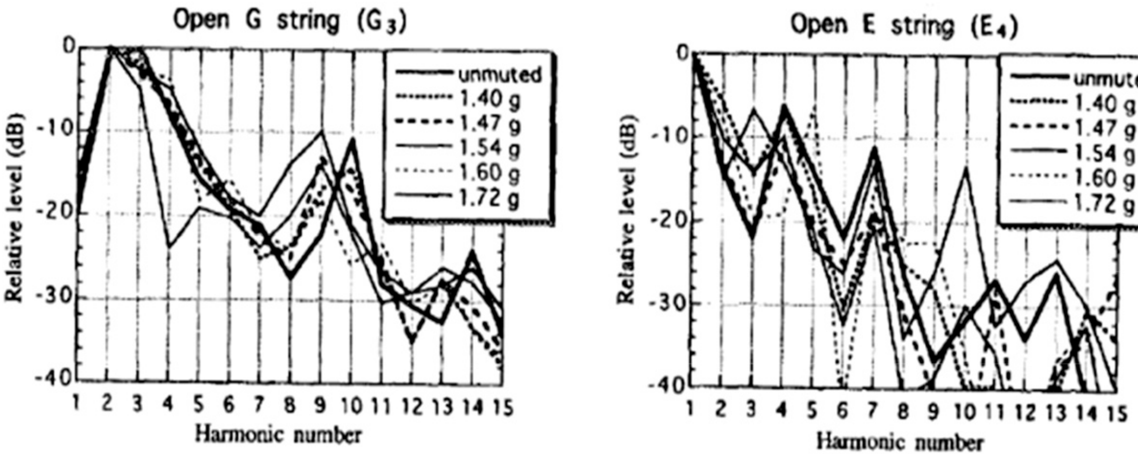


Figure 13. Specral envelopes of the muted and unmuted violin tones for ogen G(a) and E(b) strings. [3].

a small plane. There is one other consideration with respect to shrinkage. As the bridge dries out, the shrinkage of the wood will vary with the distance from the atmosphere on the edge. Bridges with pronounced profiles may be more prone to warping, particularly in cold climates where inside humidity can be very low indoors during the winter. This methodology clearly makes the core of the X thicker.

There are two ways to evaluate the profiles within the database. The first is to compare the thicknesses of the arms vs. the center thickness. The arms are a bit higher up and this will make them somewhat naturally thinner. The vertical profile can be analyzed by looking at the sequence of foot, arch, center, and top thickness.

Back-and-Forth Bridge Movements

The previous sections have concentrated on side-to-side type movements. There are movements that exist on the bridge that go backward and forward on the violin. These are referred to as "out-of-plane" vibration modes by Rodgers and Masino. They are perhaps of a little less interest, in that the sound in the violin is actually generated by side-to-side movements. That would be a bit of an oversimplification. Energy is required to excite these modes and they will absorb some energy—meaning perhaps a "hole"

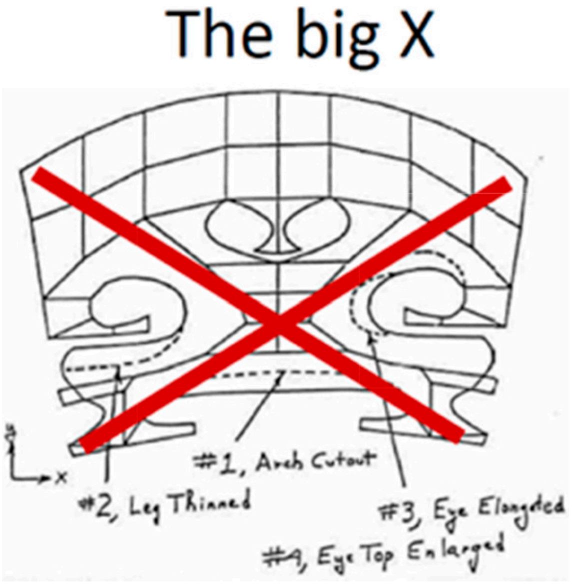


Figure 14. Bridge measurements.

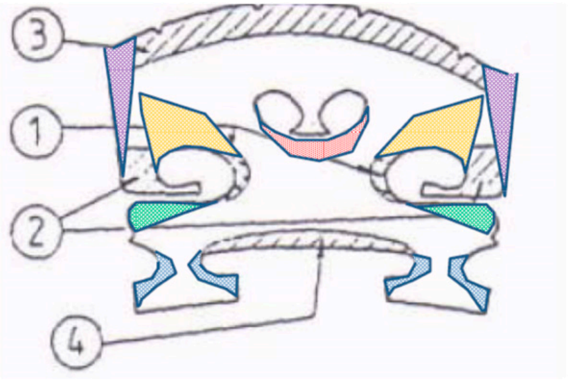


Figure 15. Bridge trimming areas [4].

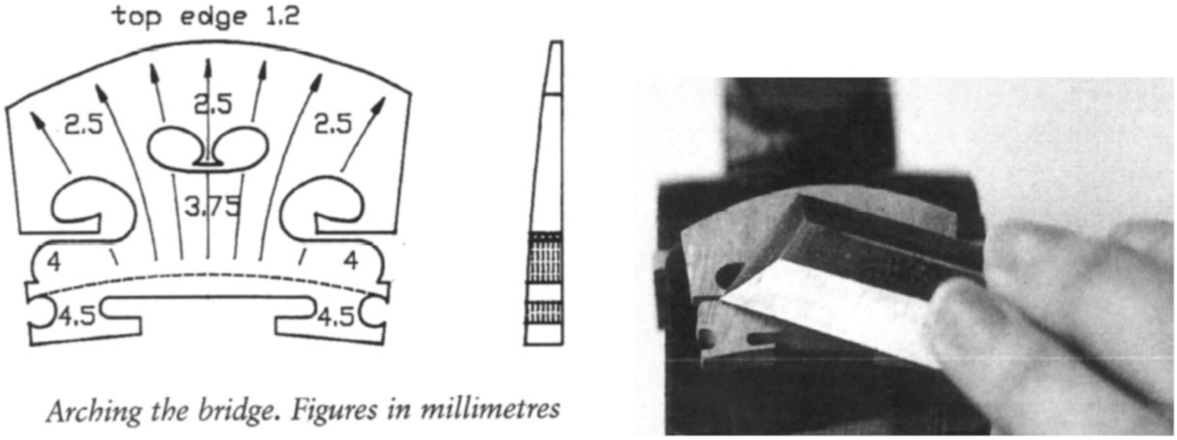


Figure 16. Bridge construction [5].

in certain frequencies. The strings at the top of the bridge will constrain back and forth movements, but some variations in string tension are likely caused as well. These modes for the violin are shown in Fig. 17.

Actual measurements of out-of-plane (back-and-forth) movements can be found in Fan and Bissinger's article [6] "*Out of Plane Violin Bridge In-Situ Motion*."

Some will involve twisting and some consist of a form of flexing forward and back. Modes for the cello are shown in Fig. 18.

From the perspective of evaluating the effects on the bridge, the fore and aft movement and twisting will be governed to some degree by thickness. The maximum bending moments in the backward and forward flexing would be in the center of the bridge—concentrated in the waist.

Bridge Tuning

Jansson (also on the Web site under References) has tried adding weights and cutting material off. Because this changes tone and each violin is different, he has suggested tuning the bridge to the instrument. He picked a Mode 1 frequency of 2.9 kHz as ideal for more than one violin. He developed a small experimental device to measure the Mode 1 frequency. One places the bridge on top of a block of aluminum. The locations that Jannson added weights to and the cutouts that he used are shown in Fig. 19: If this was found to be sufficiently promising, it might be possible to measure the bridges in the database at some point in the future.

In practical terms, buying a few blanks and trying cutting them to different weights is a practical alternative. One can also use the collective experience of luthiers (i.e., this database).

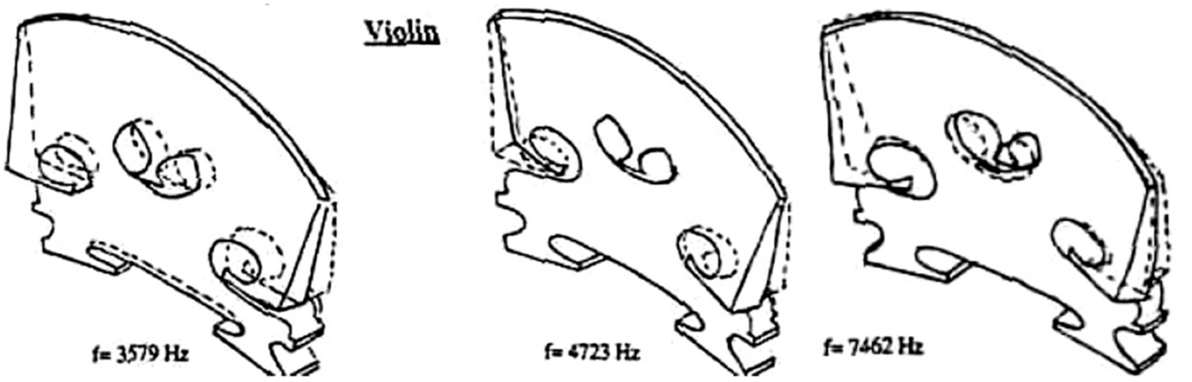


Figure 17. Bridge measurements.

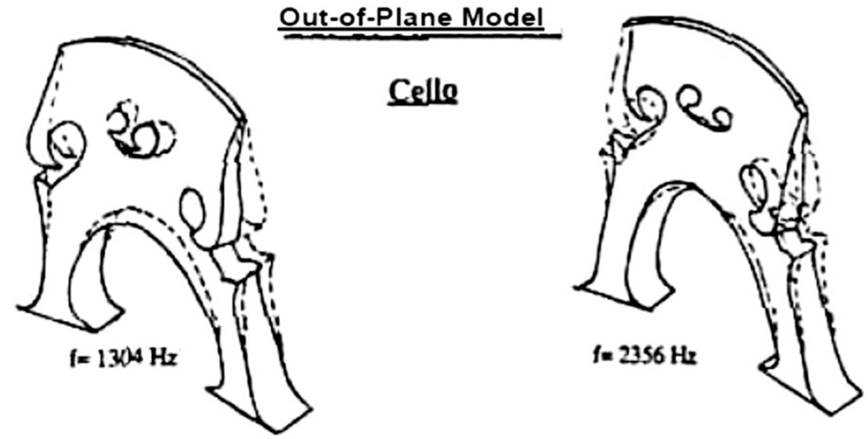


Figure 18. Bridge measurements [2].

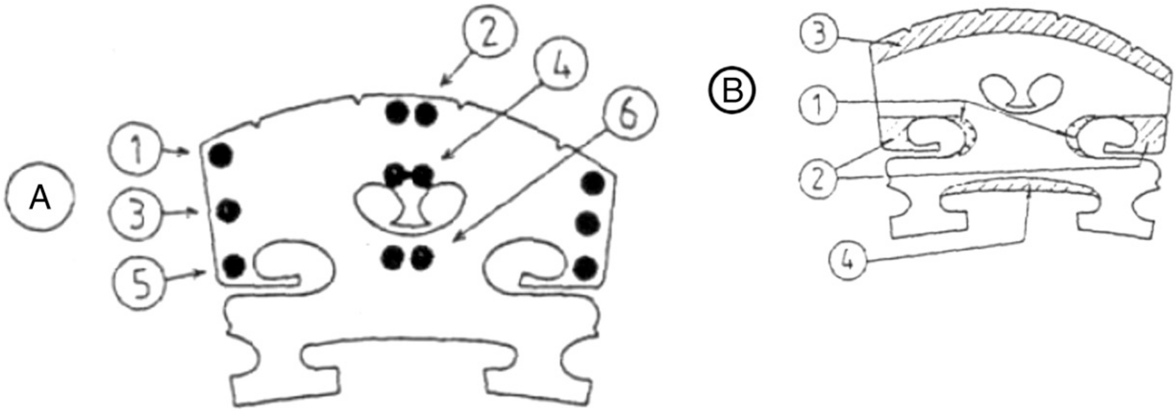


Figure 19. Sketch of violin bridge with a) positions of mass loadings, and b) positions of wood removal [4].

With a string lifter, one can switch bridges out pretty quickly and listen to the differences. This is something one can do with a client or on one's own. Blanks are actually cheap. Cutting them and fitting them does require time and effort and, therefore, cost. However, after the first bridge has been fit, one can do the subsequent fittings from a good starting point. There is also a bridge tuning setup described by Joseph Curtin's "Bridge Tuning: Methods and Equipment" [7] shown in Fig. 20 (from the VSA Papers, Summer 2005). Such devices are still rare.

The effects of bridge waist and wing mass trimming on violin radiativity were examined at the Oberlin Violin Acoustics Workshops in 2004 and 2005. The raw data were not published. However, an article was published that discussed the results: "The Violin Bridge as a Filter" [8] by George Bissinger, July 2006. From the perspective of bridge cutting, the results are not easy to discern as the article is mostly concerned with a number of other issues:

1. The article looked at 12 instruments of various qualities for a "VIOCADEAS" project. There is a huge amount of data from the large number of tests. Such a large amount of data cannot be presented in a single article. This would require a major report.
2. The main point the author is attempting to make is to shift the view of bridges from that of an isolated component to that of a system. The bridge is connected to the body of the violin that is quite a bit more massive and which is also vibrating. The bridge transfers almost all string vibration.

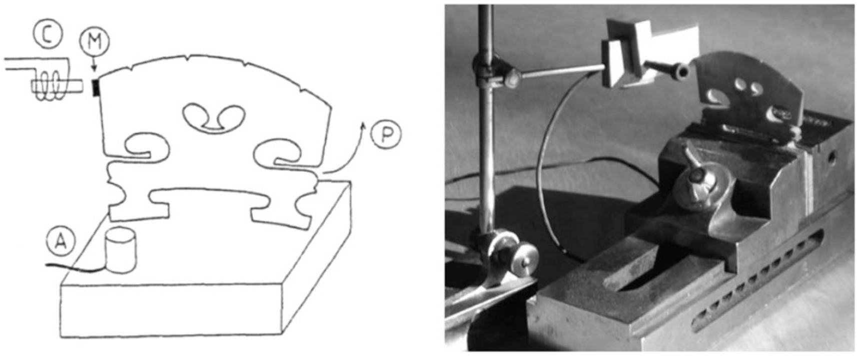


Figure 20. Bridge measurements [7].

3. Because the article is written for an academic audience, it has some distinctive features:
    a. There are detailed discussions about how the data were obtained, and this level of detail, which is necessary for an academic publication, makes getting to the "meat" quite a bit more work.
    b. The article presumes that one is familiar with some fairly detailed concepts. This requires that some background be provided to interpret the article. The two main issues are as follows:
        i. The frequency response profile and understanding what part of the frequency response profile the author is discussing;
        ii. A series of technical terms such as admittance.
    c. The article concentrates on the Mode 1 frequency change. This is a very narrow look at changes and was carried out intentionally. They wanted to change one thing and only one thing at a time. From a scientific perspective, this is good methodology.

For someone wanting to know what to do with a bridge he/she is about to cut, it is difficult to get to the "meat" quickly.

4. One of the distinctive features of violin family instruments is the frequency response at about 2300 Hz that has a very high output. Early interpretations dubbed the feature the "bridge hill." This feature is more accurately a result of the area of the top between the f holes. This is not the only article to suggest this: however, there were no changes to the bridge hill frequency when resonant properties of the bridge were changed. This was of significance in establishing a more general understanding of acoustics.
5. In the material on changes in bridges carried out at the 2004 and 2005 Oberlin Acoustics Workshop, only two instruments were evaluated: a Guarneri and a custom handmade violin. Although these data are of interest, the article discusses a narrow set of results. There were a few definitive conclusions. The three conclusions that covered all situations were as follows:

    a. Cutting the bridge had big effects on the frequency response, that is, tone and volume of the instrument. In other words, cutting the bridge is important.
    b. Thinning the wings, across the board, improved bridge performance. This is actually very helpful. Note that this was not in the Rodgers and Masino article.
    c. Thinning the waist generally resulted in a reduction of sound radiated for the A0 mode, which is the sound emitted from the f holes or body resonance at low frequencies. Cutting the waist generally reduced high-frequency response. The author's conclusion was that reducing the waist could not improve the quality of an instrument from an efficiency point of view. Top instruments have high outputs for both the AO and high-frequency bands of a frequency response curve. Clever bridge cutting is unlikely to make an instrument great.

The latter point makes sense; however, this does not mean the majority of violins cannot be improved. This part of the data was of less interest in the "big picture" and has to be looked at on a case-by-case basis.

Interaction with Top and Body

In keeping with the main theme of the Bissinger article, the next step is to look at how the bridge interacts with the rest of the violin. Two of the references included articles by Reinicke and Muller [9]. Reinicke was at the University of Berlin and Muller at Mittenwald. Reinicke recognized that the bridge vibrates in a number of different modes. Muller has written an article that not only provides a translation of Reinicke's 1973 results (in German), but also summarizes the article in more practical terms. The original research included a significant amount of math. The "story" starts with the force the bow applies to the strings and the motion that this imparts to the top of the bridge. The strings apply force at the saddle, nut, and bridge. Of these, the bridge is acknowledged as having the largest effect. This is one of the results described in Bissinger's article.

At less than 700–1000 Hz, the bridge behaves as a rigid body. This can be viewed by the force on the feet. The diagram presented in Fig. 21 was constructed by Reinicke as a proportion of downward string force that is applied to the bridge foot. Note that this was performed experimentally by fixing a bridge to a rigid plate. In essence, Rienicke was looking at what is applied to the top of the violin. Initially, at low frequencies, the force on the feet is split exactly in two—because there are two feet. However, the force becomes higher after 1000 Hz, building to a peak at a significant multiple of 0.5. The force then decreases to below 0.5 at higher frequencies. The vibration of the bridge affects the force on the feet considerably.

The data show the effect of more mass and also a change in stiffness. Stiffness was increased by shoving wedges into the edge of the wings. Note that the ratios approach a value of more than one at resonant frequencies. The stiffness changes and mass changes alter the peak frequency.

It is also possible to carry out this test on the bodies of violins. In this case, the data are described with mobility ratio. Mobility (γ) is a measure of how easily vibrations can be started and/or maintained. The velocity of the movement is divided by the applied force. Movement was directly measured on the bodies at a series of points, in addition to radiated sound from microphones. The units are, therefore, m/s/Newton. This may not be a directly intuitive unit, as a sinusoidal (vibrational) motion has a velocity that is always changing. This means some math has been used. In Fig. 22 (after Bissinger), the bass bar mobility (γ_{BB}) is divided by the sound post mobility (γ_{SP}) to give a ratio. The data show that most of the movement (velocity) per unit force is to the bass bar at low frequencies. The sound post acts like a pivot point at low frequencies—it is not moving as much. At higher frequencies, over about

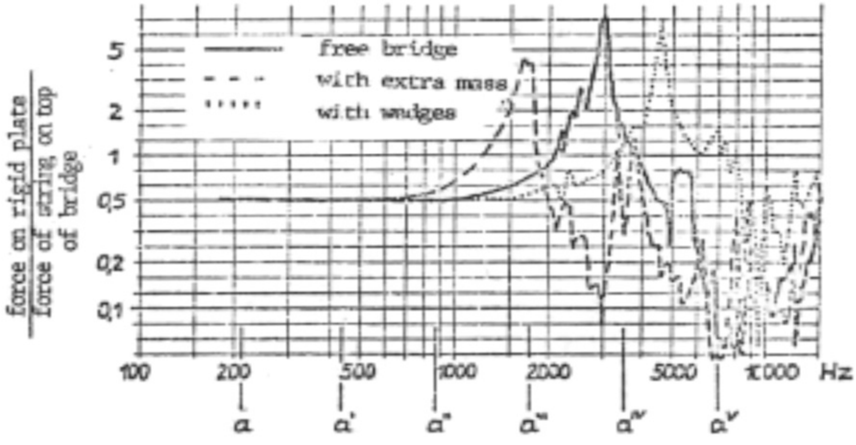


Figure 21. Force transfer by the bridge [9].

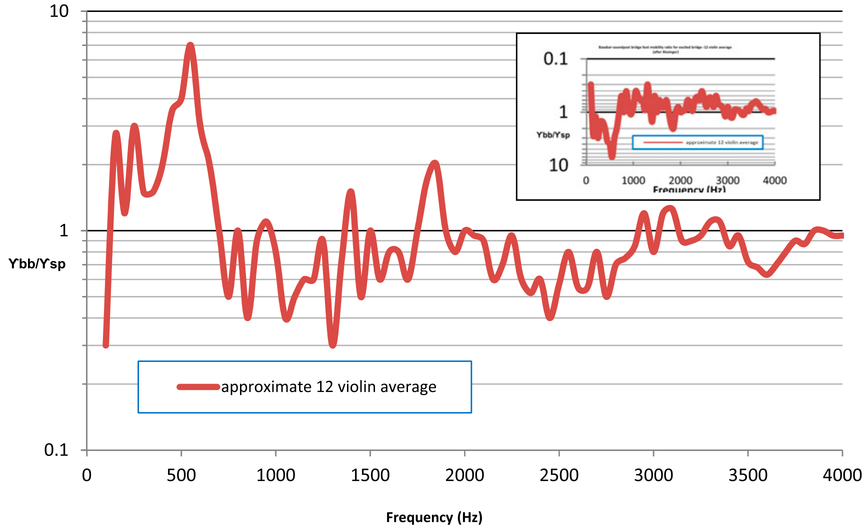


Figure 22. Basebar-soundpost bridge foot mobility ratio for excited bridge; 12 violin average [8].

700 Hz, the situation generally reverses. Both feet of the bridge are still moving; however, the SP foot is generally moving more per unit force. The diagram has been inverted to show this in the inset. The response has become more complex.

The graph shows a similar shape to the Reinicke graph of force at the feet. The frequencies are different and the peak is not quite as pronounced; however, the transition from rigid body to bridge vibration modes is clearly evident.

The up-and-down and the back-and-forth movements of the bridge can be seen as an energy loss (Bissinger) by the gap between the red line and the blue line (with markers) in the graph in Fig. 23. The up-and-down motion corresponds with the “Squat” mode. These data are from the VIOCDEAS project (after Bissinger).

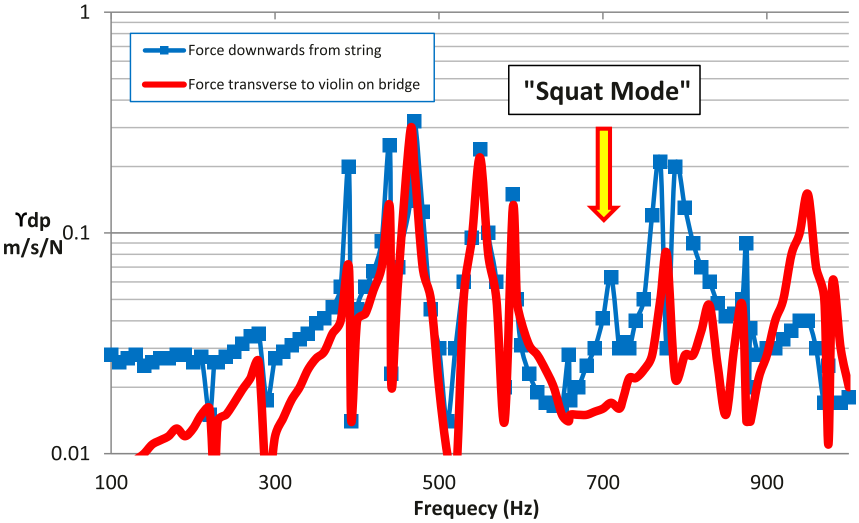


Figure 23. Mobility resulting from excitation force parallel to, and at right angles to bridge [8].

Arch Height

Muller's article gives some additional insight. His analysis uses relative changes, so it does not represent a true frequency response curve. He addresses specifically arching on the bottom of the bridge. These data are different in that it deals with sound pressure levels in dB.

From Fig. 24, the increase in arch increases sound volume below about 3000 Hz and creates a valley and a peak between 3000 Hz and 5000 Hz. The high end above 5000 Hz is reduced. Differences of 1 or 2 dB are generally audible. So these are noticeable effects.

Instrument Variation

He also provides an interesting comparison of different instruments (Fig. 25).

The translation of thin and thick walled is likely a bit of a mistranslation. The thin-walled violin is 1.5 mm thick, and the thick-walled violin is 3.0 mm. These are not rib thicknesses: the author is clearly referring to average top thickness. The normal thickness is not specifically

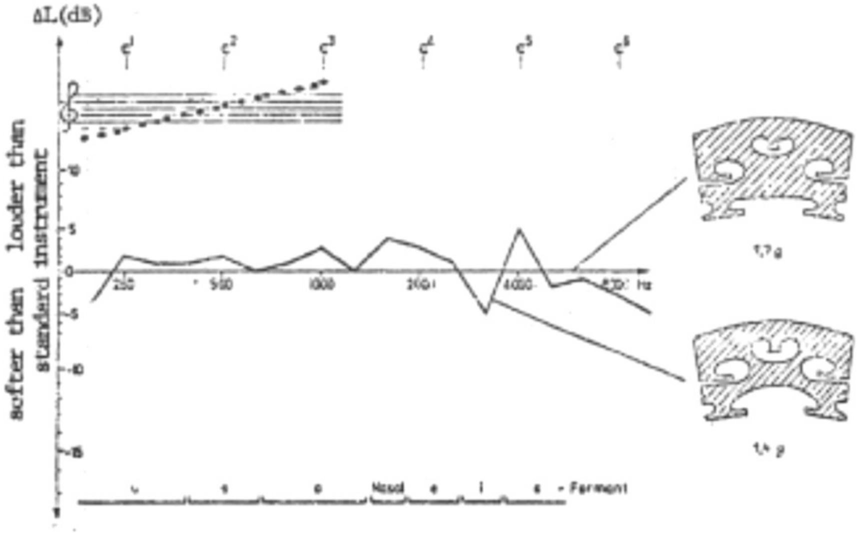


Figure 24. Changes in bridge transmission caused by extreme trimming [9].

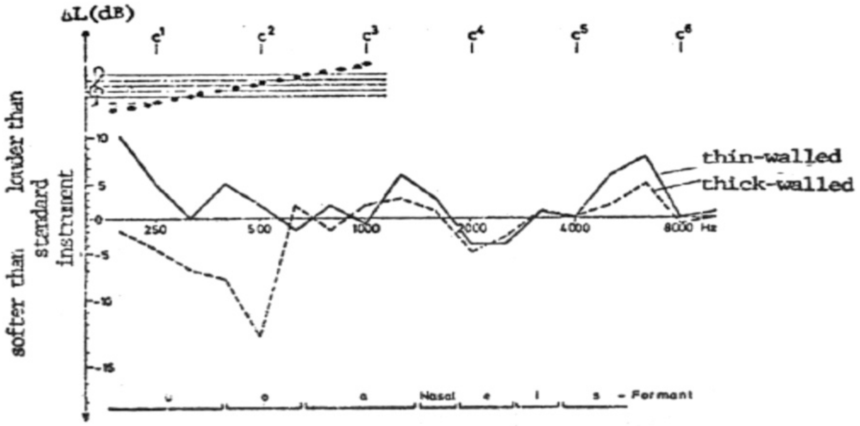


Figure 25. Comparison of thick-walled and thin-walled violins with a normal instrument [9].

mentioned; it is described as a good but not great instrument. This would typically be in the 2- to 2.5-mm range. The thin top gives better bass and higher trebles, and the thick top results in a loss of bass volume, relative to a normal top graduation.

F-Hole Notches

Another point made by Muller, which seems to be frequently overlooked by many, is the position of the bridge along the axis of the strings. One can move the bridge feet back and forth, and the top of the bridge. Most people regard the notches in the f hole as a strict position rather than a guide. This is demonstrated in Fig. 26, wherein the middle frequencies are enhanced.

Bridge Reengineering

One of the diagrams in Fig. 27 discusses the changes in bridge engineering that occurred with the addition of bridge kidneys and heart.

Although this follows the lines of reengineering the bridge, the effects of different cuts can be seen. The first addition of the kidney changes the waist and the Mode 1 resonant frequency. Note that this is all compared with a finished

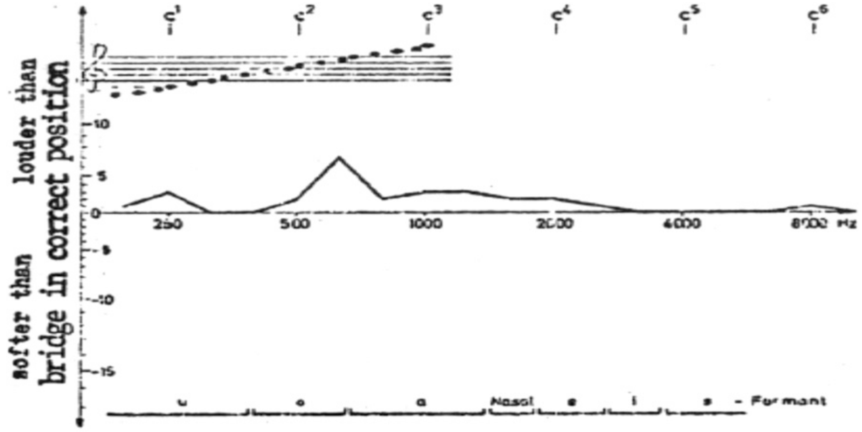


Figure 26. Changes in bridge transmission caused by a shift in position [9].

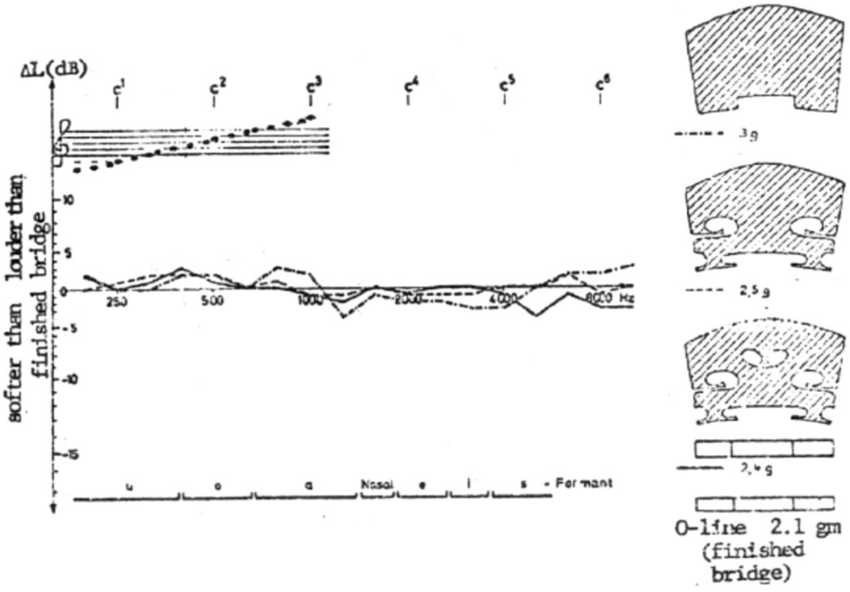


Figure 27. Vibration transfer as a function of the condition of the bridge [9].

bridge, and the weights are provided. The final bridge of 2.1 g would be considered a normal bridge weight, although this is slightly higher than the average weight. It follows that different gradations of cut might provide variations.

Waist Trims

Bissinger's article examines waist reduction, which changes Mode 1 frequency. Waist width trimming had more effect than wing trimming. The results of waist trimming are shown in Figs. 28 and 29.

The lines with solid circles and solid lines represent the higher mode frequency, i.e., no waist trimming. The dashed line, with no markers, represents the trimmed waists. No additional trimming gives better performance for the two violins tested. Note that thickness is not outlined, nor is the actual mass or waist width provided. The waist may already have been trimmed. Perhaps less reduction would be ideal. The article does not suggest abandoning the kidneys, which creates the waist or a custom wider waisted blank.

Wing Trimming

The effect of wing weight reductions was uniformly more volume produced. Radiativity is the sound pressure level per unit force applied to the bridge (with the hammer). Note that this is a limited sample set, with only two violins (again after Bissinger). The Alf violin is a modern maker.

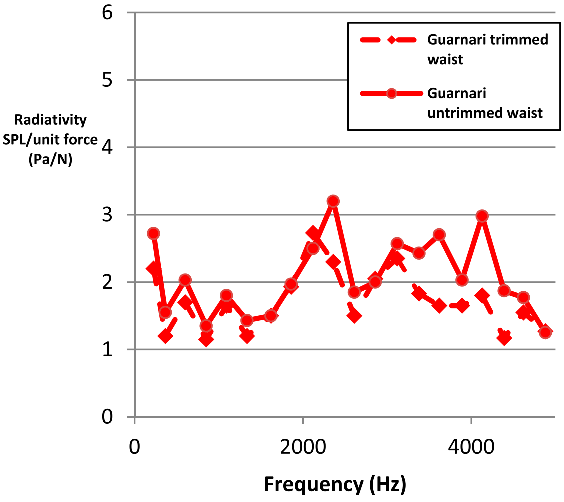


Figure 28. Variation of radiativity for Guaneri with waist trimming vs. frequency [8].

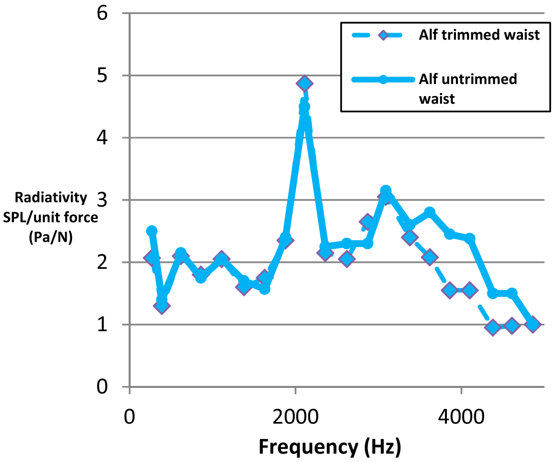


Figure 29. Variation of radiativity for Alf with waist trimming vs. frequency [8].

With the exception of a couple of points, the solid line, trimmed wings, produces more sound than the untrimmed wings (Fig. 30).

This was one of the definitive conclusions from Bissinger's article. Over-trimming the bridge on the Guarneri made the resultant frequency response profile similar to that of a very ordinary violin.

In the Oberlin tests, it was possible to greatly affect the modern violin to more closely match the historical "target" shown with the dark line. In this case, the data are expressed as a ratio of radiated sound to that of the 1300- to 1640-Hz band. These are not absolute sound levels. In this case, the bass has been improved and the treble reduced with the right bridge trim (from Bissinger). The light blue curves in Fig. 31 show an envelope of 20 different bridge variations. Twenty different curves make a rather messy graph; hence, the upper and lower bounds have been shown and error bands were removed.

In the case at hand, the bridge mass reductions reduce performance; however, the base or starting point is described only as a Mode 1 frequency, and usable data, such as mass and dimensions, are not given. The optimum was 3.6 kHz, instead of the 2.9 kHz that Jansson selected. Both the Reinicke and Bissinger articles conclude that changes must be made for the particular violin at hand. This would be consistent with these observations.

The last graph probably represents the ideal. Different bridges are tested with a frequency

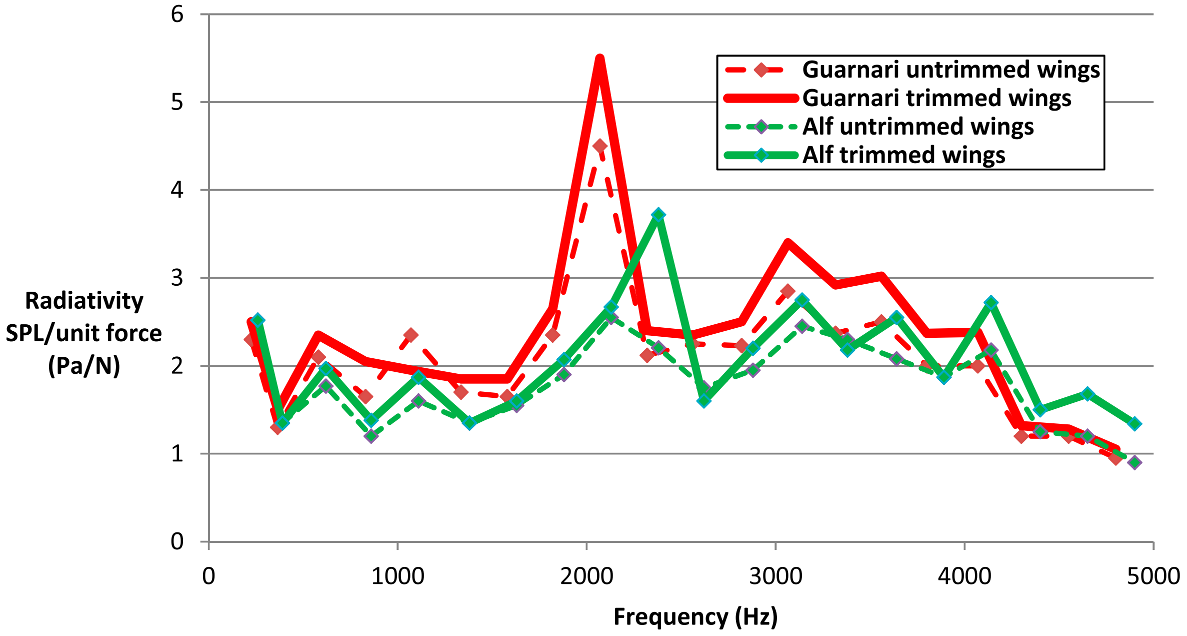


Figure 30. Variation of radiativity from wing trimming vs. frequency [8].

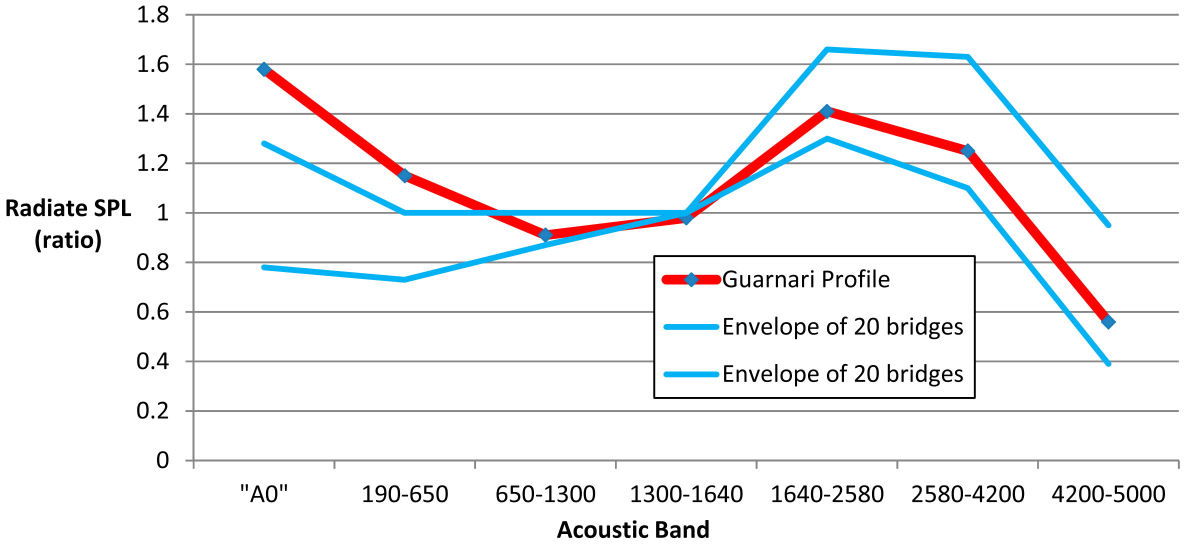


Figure 31. Radiation variation vs. frequency for different bridge cuts [8].

response profile, and the bridge closest to the target is selected. (It is not clear whether the ideal target is known.) This suggests making a series of bridge blanks and either quantitatively testing them, or doing a double blind test with listeners.

Evaluating the Dimensions of Bridges

Now that some thought has been given to what the data might affect, it is time to look at the actual dimensions of bridges. The examples were screened for workmanship and make. The data should give an indication of what the norms are in bridge construction. The database is overall quite large. The number of bridge data points is shown in the following table. There are also good sized samples of many prominent luthiers in the database, and this gives an opportunity to compare how their approaches vary.

	Maximum Number of Points	Minimum Number of Points
Violin	767	310
Viola	121	45
Cello	174	58

There are some key elements that are missing: There is no quantitative indication of how the bridges sound and the interaction between the bridge and instrument cannot be captured. Suffice to say that the results need to be interpreted in this context.

Bridge as a Filter

Muller states: "The bridge serves as a sort of tone filter, strongly attenuating certain frequency ranges and letting others through un-weakened." This could be viewed as creating distinctive concentrations.

VIOLIN

Data Screening

For the most part, very unusual bridges have been removed from the statistics. They can be viewed on the Web site. The intent of this study was to provide good statistics on conventional bridges.

Mass

Mass has a big impact on sound. Raw bridges range about 2.75–3.5 g in weight. The data indicate there are a few luthiers who do not thin them at all. At the other end, there are not very many weighing less than 1.5 g. There are a few between 1 and 1.5 g. So what is the lower limit? A general reluctance to go below 1.5 g is evident. But the data suggest this is possible. These are mostly used bridges from the pictures and this means that they survived use. Practical experience shows that lighter bridges less than 2 g can improve volume and tone.

In Fig. 32, there are two points where thicker than normal blanks are evident. Interestingly, there is a bridge in the data that has four feet in a patented design.

The average mass is 2.00 g, with a standard deviation of 0.26 g (13%), a maximum of 3.0 g, and a minimum of 0.8 g. The last point was checked and it would appear to be a very thin bridge throughout that was used to get to such a low weight. It had very narrow feet.

Thickness at the Top

The thickness at the top seems to be governed by providing adequate thickness to support the strings and the stress concentrations that exist

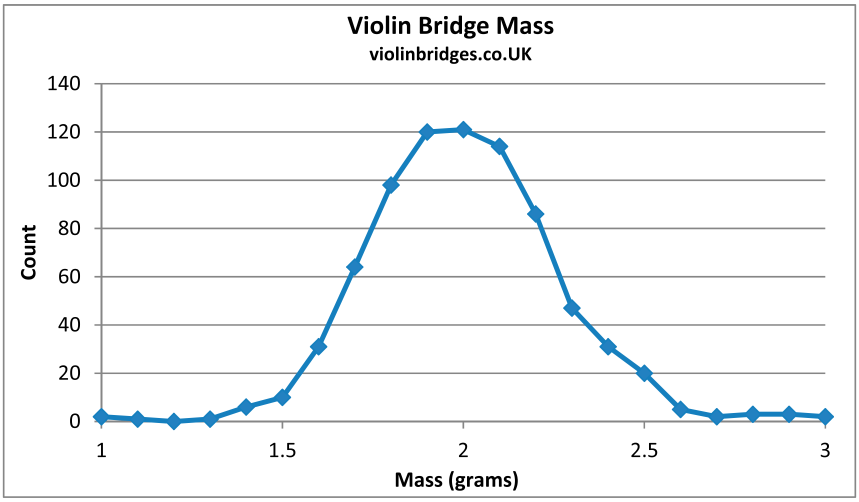


Figure 32. Distribution of bridge mass for violin dataset.

there. The distribution of thickness is shown in Fig. 33.

There is almost no correlation between bridge mass and top thickness. Thickness at the top does affect durability. Although the lower limit is about 0.9 mm, many shops would use 1.2 or 1.3 as a lower limit, particularly with student violins. This is to prevent damage from the strings. The E string has the highest stress concentration. The distribution is not symmetric about the peak of 1.3 mm; it is skewed slightly to the thicker side.

The graph includes a value for R^2, which is a statistical measure of how good the correlation is. The R^2 shown is approximately 0.03. To put this in context, the R^2 for height vs. weight in people correlates at 0.6. A defined relation usually results in an R^2 of 0.8 or higher. Correlations of 1.0 do not usually exist because of measurement variations. Values between 0.95 and 0.99 do exist in practice.

The data in Fig. 34 shows there is no correlation between bridge mass and thickness at the top. In practical terms, if the top is too thin, it becomes fragile. So, sticking to higher than 1.0 mm on a violin is a good idea. There are the odd violin bridges that drop below 1.0 mm. This was perhaps accidental. In any event, there are

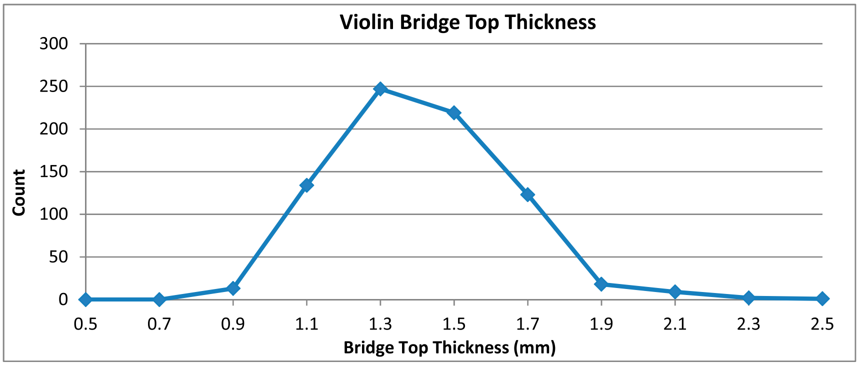


Figure 33. Distribution of bridge top thickness for violin dataset.

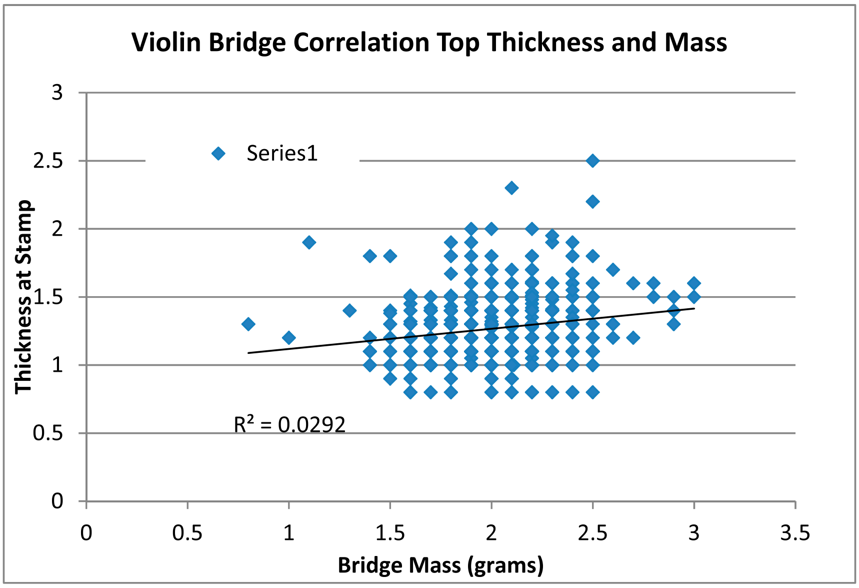


Figure 34. Correlation between violin bridge top thickness and violin bridge mass.

enough data to suggest that it is possible, and dropping slightly below 1.0 mm does not automatically require starting over and throwing the bridge out. There are a few bridges where the top thickness appears to be the same thickness as a raw blank at 3.0 mm.

On the cello bridges, there were a few bridges that dropped below 2.0 mm, and this is also rare. In one case, a thin bridge showed clear signs of damage from the strings digging into the top of the bridge.

Foot Thickness

Starting from the bottom up, foot thickness is where the body of the instrument and the bridge meet. Stress concentration on the top is a concern as spruce used in tops is considerably softer than the maple used in bridges. From Fig. 35, it seems that it is rare to go below 4.0 mm. The data show some bridges where the bridge blank does not appear to have been reduced considerably. Two handy Despiau and Aubert bridges show raw bridge blanks with 5.5- and 5.35-mm-thick feet. Note that this is not statistically valid. There is one that is quite thick and this appears to be a custom blank. I have seen one well-known luthier who insets small hardwood pads on the top of the violin for protection.

The average foot thickness is 4.43 mm, with a standard deviation of 0.34 mm (7.7%). The minimum was 3.5 mm and the maximum was 6.5 mm.

Foot Width

The left and right foot width was measured, and the results presented in Fig. 36.

The distribution looks a little skewed to the short side. It would appear that blanks come mostly at about 12 mm and are shortened by some luthiers. Values lesser than 10 and greater than 13 are uncommon. The averages for the left and right foot width are 11.54 and 11.58 mm, respectively, with standard deviations of 0.8 and 0.76 mm (7.0% and 6.6%), respectively. The maximum is 14.2 mm and the minimum is 8 mm.

Ankle Width

The next data presented is of ankle width (Fig. 37). Ankles average 3.51 mm for the LHS and 3.50 for the RHS. The standard deviation is 0.64 and 0.63 mm (18.2% and 18.0%), respectively. The maximum is 7.0 and the minimum is 1.8 mm.

The distribution is very similar from the left to the right. Note that blanks are typically about 5 mm wide at the feet.

Thigh Width

Thigh width is affected by both increasing the arch and expanding the bottom part of the kidneys. More variation is, therefore, expected in this measurement.

The handy Despiau and Aubert blanks are 7.15 and 6.85 mm, respectively (so this is not statistically valid). The data in Fig. 38 suggest

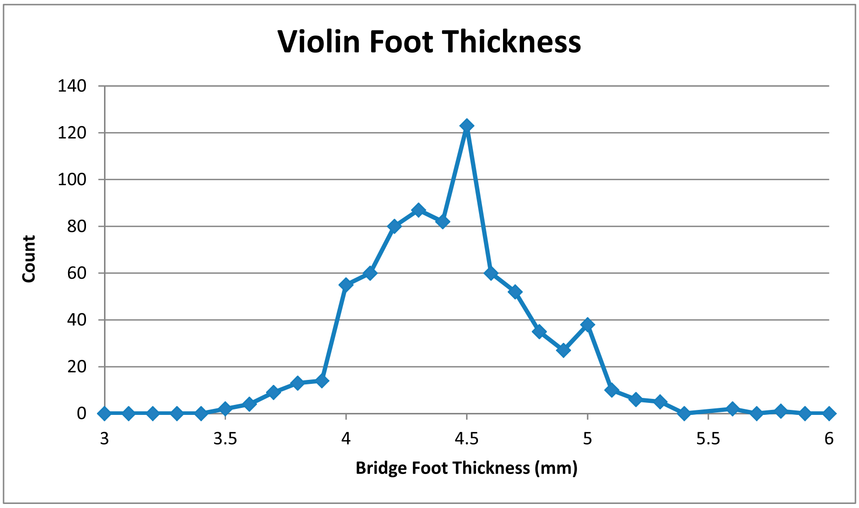


Figure 35. Distribution of foot thickness for violin bridge data set.

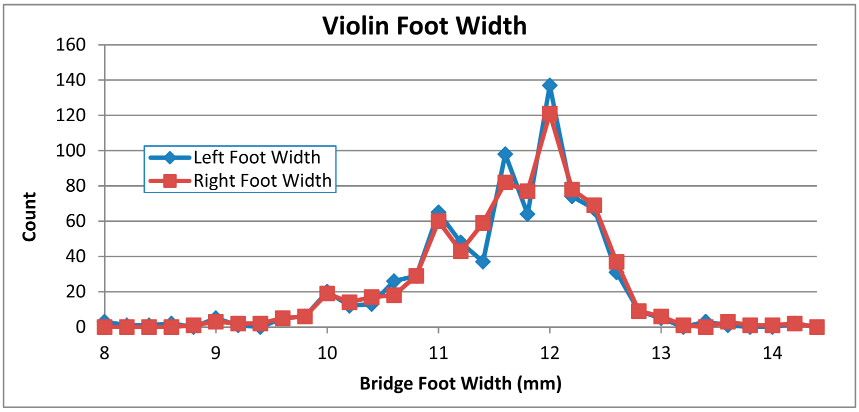


Figure 36. Distribution of foot width for violin bridge data set.

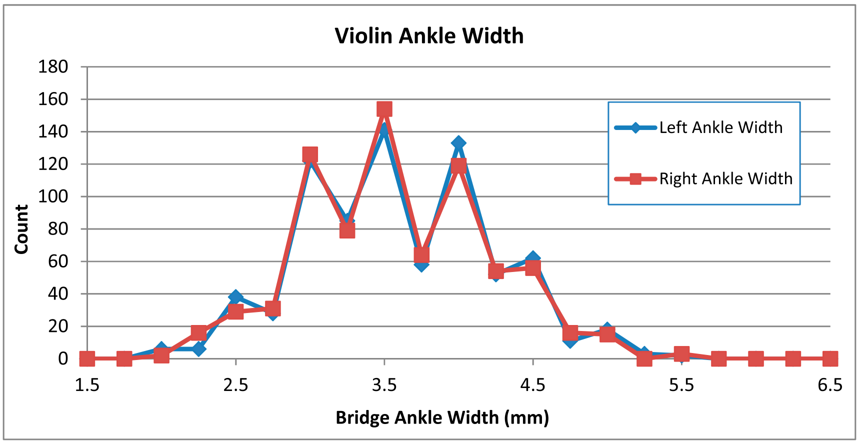


Figure 37. Distribution of bridge ankle widths for violin data set.

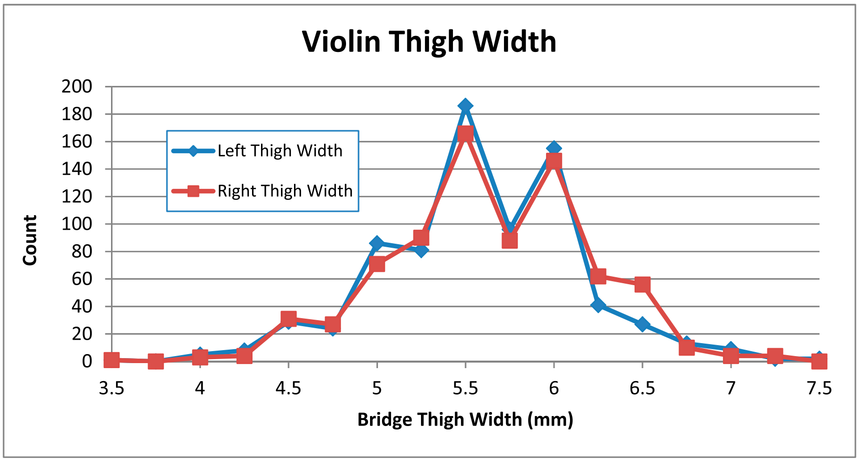


Figure 38. Distribution of bridge thigh widths for violin data set.

that the thighs get altered in general. The average is 5.5 and 5.54 mm for the left and right, respectively, with standard deviations of 0.55 and 0.58 mm (10.1% and 10.5%), respectively. The maximum is 7.3 and 8.0 mm for the left and right, respectively, and the minimum is 3.1 and 2.5 mm for the left and right thighs, respectively. The lowest common data are about 4.5 mm.

Arm Width

Arm width data are similar to the thigh data, but do not show as much variation from the left to right (Fig. 39). The handy blanks show arm widths of 6.15 and 6.6 mm. This would indicate that some cutting via enlargement of the kidneys or heart occurs. Averages of the left and right are 5.27 and 5.26 mm, respectively, with standard deviations of 0.53 and 0.53 mm (10.1% and 10.0%—note that the 0.53 mm is rounded), respectively. The maximum for the left and right were 7.30 and 7.42 mm, respectively, and the minimum for the left and right were 3.2 and 3.5 mm, respectively.

Balance

Evidence was evaluated to see whether luthiers adjusted balance by changing the thickness of the arms and thighs. It appears this is not generally the case, although this does not mean it was not practiced. Of course, moving the SP is probably the first-order change. Figure 40 suggests this does not happen very often. The left/right is a ratio of the left over the right arm or thigh thickness. Probably, a 10% (0.9 and 1.1 on graph) variation is just because most of this cutting is carried out by eye. One would suspect that some effort has been made to change the sound balance. The statistics suggest this does happen, although this could be attributed to accident.

Waist Width

As previously outlined, the waist affects the overall stiffness of the bridge in both width and thickness. Some variation is, therefore, expected. The waist can be narrowed by cutting out the inside edges of the kidneys, from the left and right.

The waist does not show a great deal of variation. The handy blanks show waist widths of 16.5 and 17.3 mm. There are clearly wider blanks available. Some cutting via enlargement of the kidneys occurs. From Fig. 41, the average waist is 15.97 mm thick, indicating not a lot of narrowing. The standard deviation is 1.12 mm or 7.0%, less than that for the arms and legs on a percent basis, but larger on an absolute basis. This suggests the average cut is on the order of 17 – 16 = 1.0 mm or 0.5 mm per side. The general upper limit is about 17 – 14.5 = 2.5 mm or 1.25 mm per side. The maximum waist width is 21 mm and the minimum is 11.71 mm. The latter would be about 17 – 12 = 5 mm or about 2.5 mm per side. Thickness is important too, and this is addressed later. Note that fractional sizes were filtered based on bridge width.

Violin Bridge Width

Bridge width data are shown in Fig. 42.

The average width is 40.81 mm, with a standard deviation of 1.01 mm or 2.5%. This

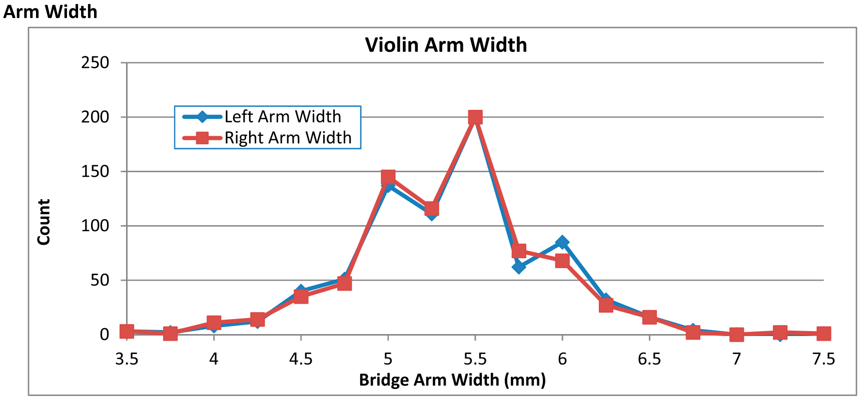


Figure 39. Distribution of bridge arm width for violin data set.

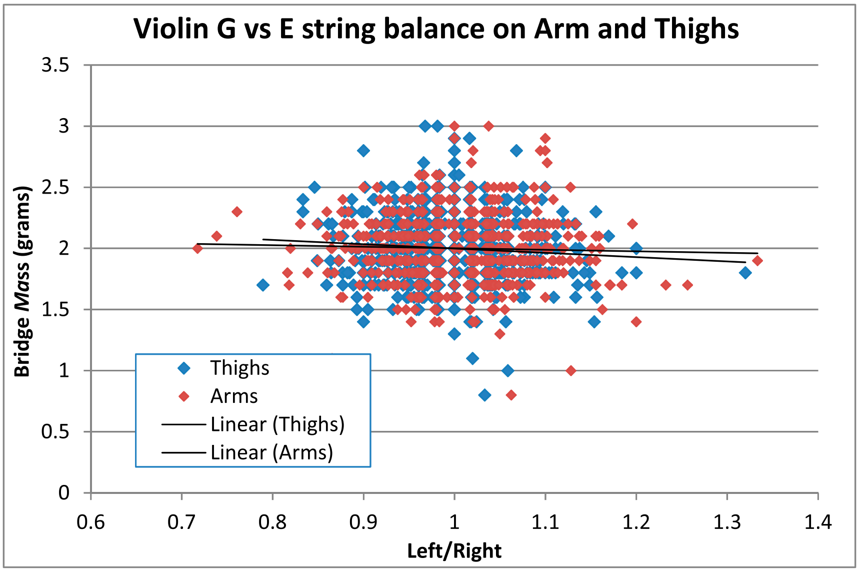


Figure 40. Ratio of arms and thigh widths (G side to E side) for violins (1 = same on both sides).

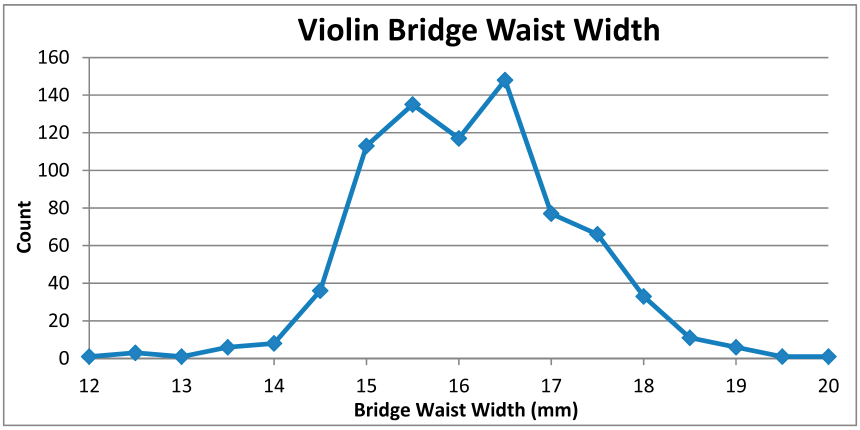


Figure 41. Distribution of bridge waist width for violin data set.

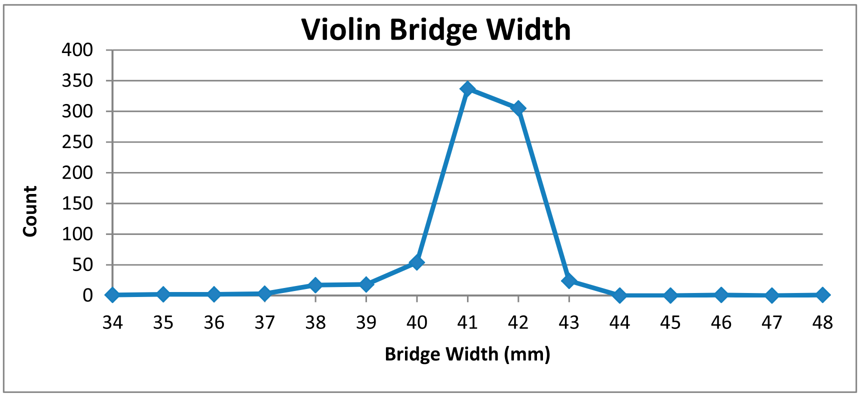


Figure 42. Distribution of bridge width for violin data set.

is a very small variation. The maximum is 47.5 mm and the minimum is 34 mm. The width of 47.5 mm looks like a viola bridge; however, the instrument is listed as a violin. It is a hand-cut custom bridge of conventional design.

Reducing Mass

Evidence was also examined as to how luthiers would reduce weight. General thickness seems to be part of it. Note that the degree of cutout is also important. So some statistics were run to see how mass would be systematically reduced. The first variation was on the top width (Fig. 43).

There seems to be no correlation here at all. The next correlation was with thickness (Fig. 44).

This shows that the mass is related to thickness but the correlation is well below 0.6 (height and weight in people). The R^2 of 0.25 for arch thickness is not that strong. Thickness clearly counts, but it seems that cutout matters as well. The thickness at the foot and center counts somewhat less. The foot width is understandable as the feet are narrow and do not have a lot of mass. The center is more surprising. The next check would be thigh width (Fig. 45).

Data for arm width are presented in Fig. 46.

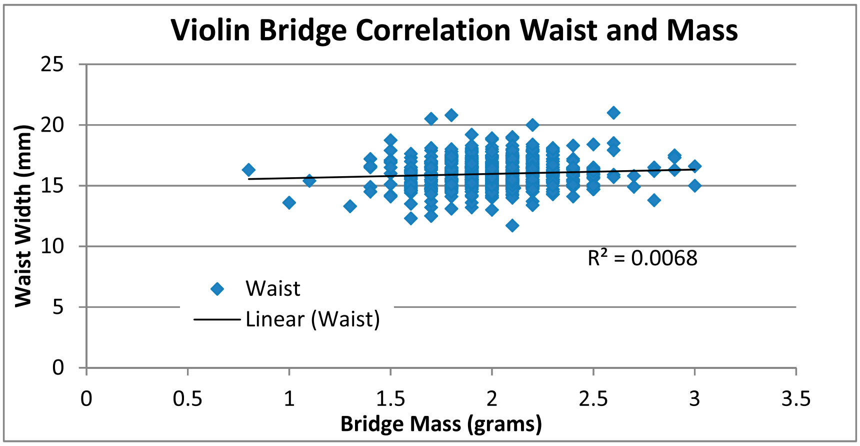


Figure 43. Correlation between bridge waist width and bridge mass for violins.

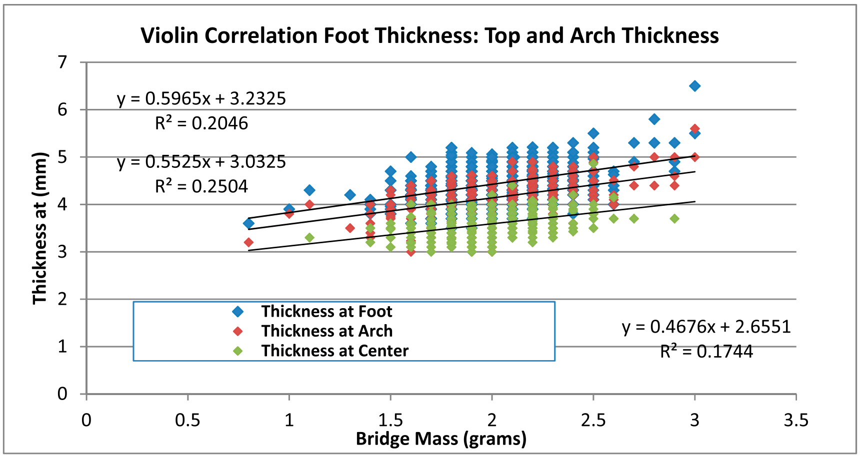


Figure 44. Correlation between bridge foot thickness, top thickness and arch thickness for violins.

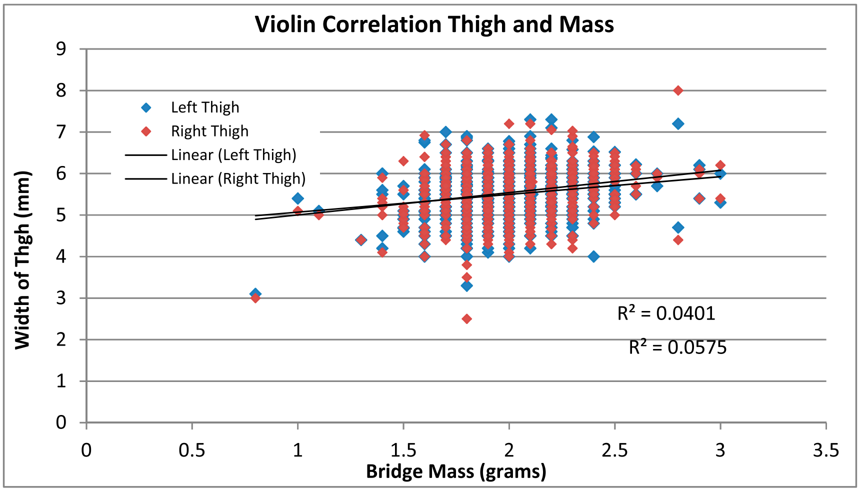


Figure 45. Correlation between thigh width and bridge mass for violin data set.

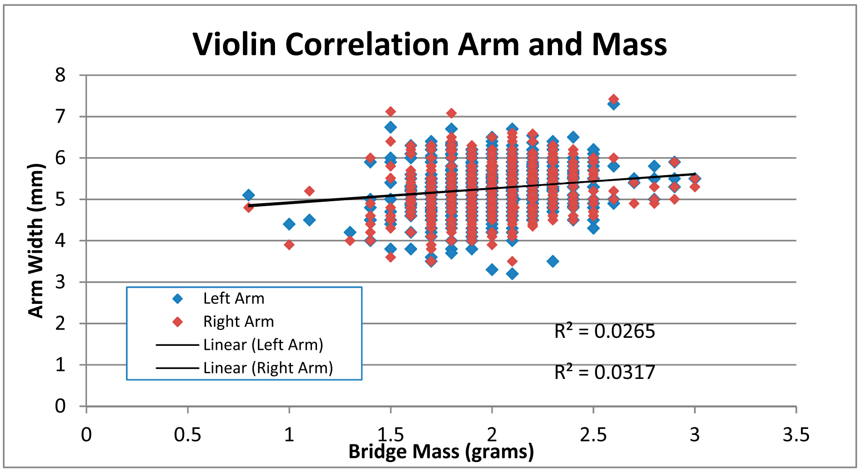


Figure 46. Correlation between arm thickness and bridge mass for violin data set.

Arm, thigh, and ankle width would all count, but with an R^2 of between 0.03 and 0.06, these do not seem to individually contribute significantly. A correlation was also generated of how mass varied with waist width, as shown in Fig. 47.

This amounts to almost no correlation at all. The effect of ankle width was also examined; however, ankles are pretty thin and would not be expected to have a big effect. This is indeed the case as presented in Fig. 48.

Density data are not provided and they will vary. The data obtained do not track arch cutout or changes to feet and hips. One other item would be parchment. All of ankle, thigh, arm, and waist width seem to correlate to about the same degree. In other words, it would appear that once the decision has been made to reduce mass, that changes may be made relatively uniformly.

It was decided to try and capture the vertical profile. This did not work easily as some 300 plus profile created a graph that looks a lot like a plate of spaghetti. The data were sorted on mass from the lowest to highest and then divided into 10 groups of approximately 15 data

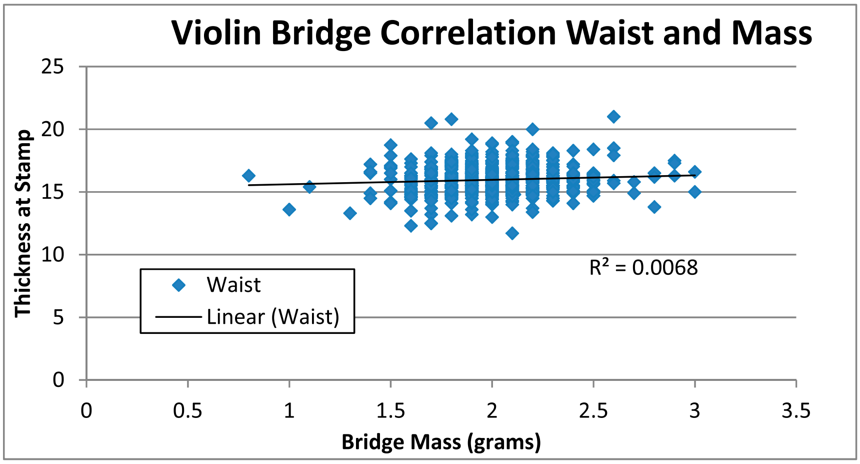


Figure 47. Correlation between bridge waist width and bridge mass for violins.

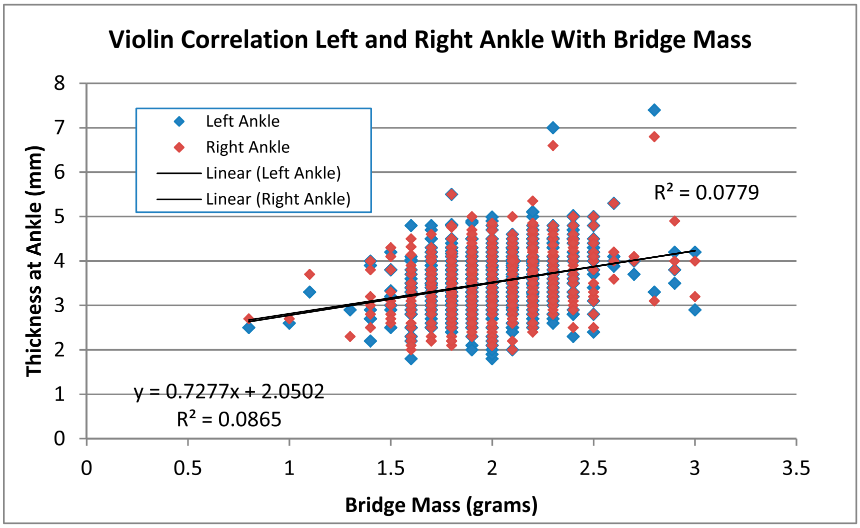


Figure 48. Correlation between left and right ankle width with bridge mass for violins.

sets. The profiles were then averaged and plotted (Fig. 49).

The exact distance above the feet is not exactly the same for each bridge or group of bridges. So some license has been taken. In general however, it is clear that thicker bridges are heavier. The profile is not linear from the bridge foot to bridge top and corresponds with the profile described earlier. Although the author has seen letter A profiles on good quality violins, they are not evident in the data set.

Some further analysis was constructed. Using the aforementioned profiles, the volume of the bridge could be estimated by dividing the mass by the density of maple at approximately 0.65 g/cc. The aforementioned profiles can be used to calculate average thickness with mass. The area can be calculated by dividing volume by average thickness.

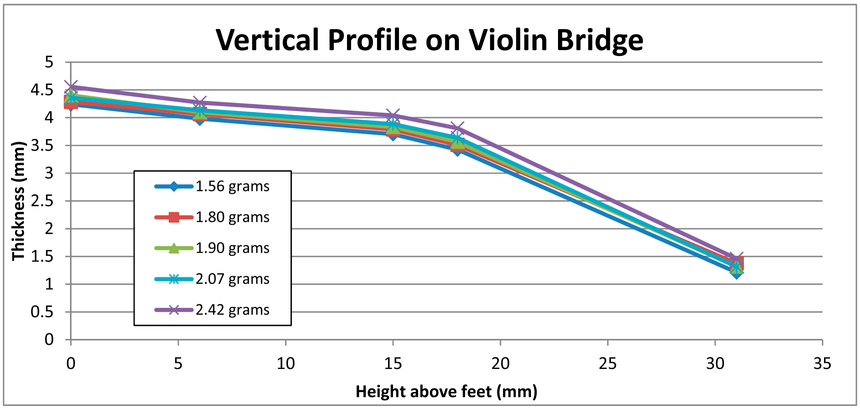


Figure 49. Vertical thickness distribution for violin bridges.

Mass	Volume	Average Thickness	Area	Thickness Reduced	Area Reduced
1.56	2.4051	3.159	7.61	–9.26	–28.94%
1.8	2.7692	3.2625	8.49	–6.29	–20.73%
1.9	2.923	3.2813	8.91	–5.74	–16.81%
2.07	3.1846	3.32	9.59	–4.63	–10.46%
2.42	3.7282	3.4813	10.71	0.00	0.00

In this calculation, it is assumed that the minimum bridge fitting is represented by the 2.42-g grouping. This includes cutting the profile on the top, fitting the feet, and thinning the top enough to get close to 1.1 or 1.2 mm. As the weight is reduced, the effective area steadily decreases. The area of the bridge varies considerably more—by almost a 30% reduction. The bridge thickness does not change that much, less than ten percent. From this, one may conclude that lighter bridges are achieved by some thinning, but more importantly by the extent of cutouts that reduce the bridge area by up to 30%.

A few comments are in order for the graph in Fig. 50. The R^2 on bridge volume vs. mass is 1 because it is calculated—there are no measurement errors. Recall that the density is assumed and does not vary. The area change does not

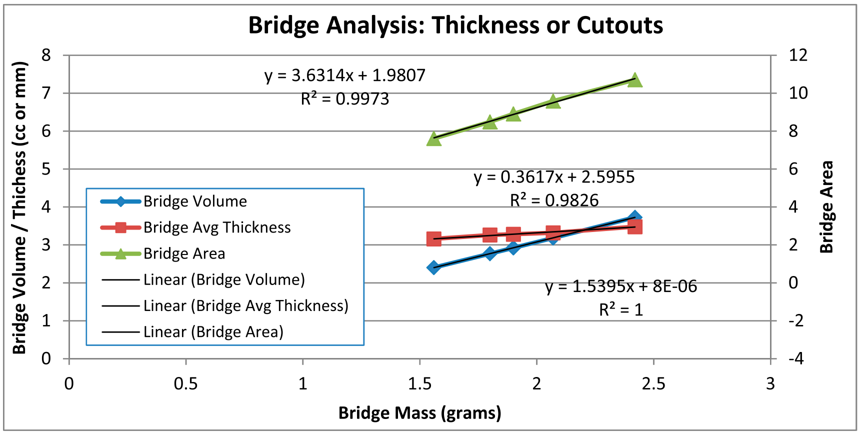


Figure 50. Correlation of bridge thickness and area for violin data set.

directly correlate to arm and thigh width, or waist width. The calculation ignores that thinning can be applied to the edges of the bridge. This will reduce the effect of cutouts on mass somewhat.

Transverse Thinning

As shown previously in the section on bridge construction technique, there is a thinning toward the edges on the upper half of the bridge. One measure of this is the thickness of the center compared with that of the arms on either side. This falls somewhat short of measuring the thickness at the edges and does not give a quantitative profile as shown earlier. The center thickness was divided by the average arm thickness to indicate how much thicker the center was compared with the arms. One was subtracted and it was then multiplied by a hundred to indicate the percent increase in thickness between the arms and the center. The data are shown in Fig. 51. Overall, it can be seen that the center is on average 19.7% thicker than the arms, with a standard deviation of 6.88%. The maximum thickness difference was 54% and the minimum 1.6%. The latter bridge would be flat across. Very few bridges are this flat and would correspond to an A profile. The distribution of data is shown in Fig. 52. This indicates the effects of cutouts should be downgraded from 30%. The arms are about 15% thinner; however, this is applied to the top half and the edges will be thinner yet. So, a 15% reduction due to transverse thinning is probably in the correct range. This leaves about 15% reduction in area due to cutouts.

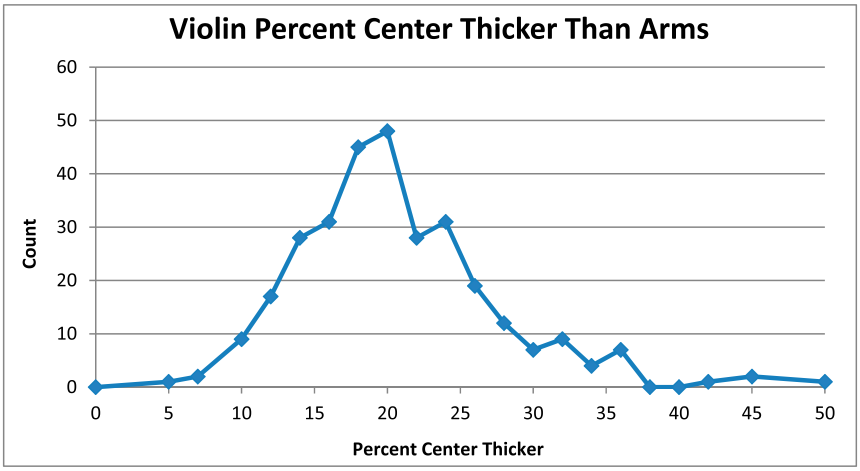


Figure 51. Lateral variation in thickness on violin bridges bridge center as percent of arm thickness.

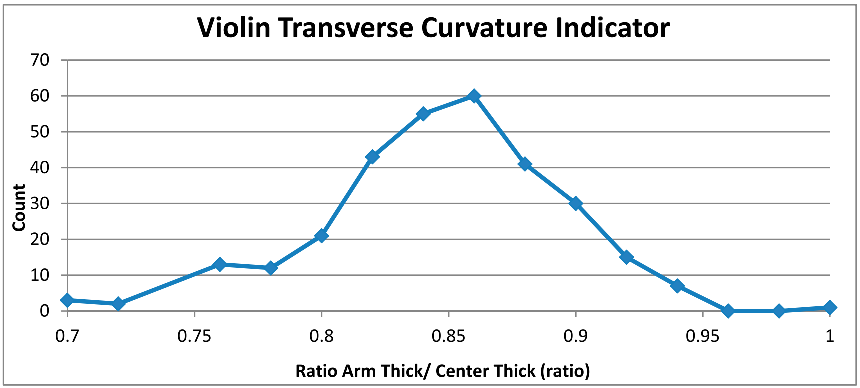


Figure 52. Indicator of transverse curvature–arm thickness as fraction of center thickness.

Note that the total weight reduction on bridges is significant. Using some handy bridge blanks, the Despiau are heavier—averaging about 3.73 g. The Auberts are a bit lighter at 2.873 g. At 2 g for final bridge weight, that is 46% for the Despiau and 30% for the Aubert end up on the floor. A comment on bottom arch is also worthwhile. A fairly aggressive cut would take off a couple of millimeters across, about 23 mm, which comes to 0.20 × 0.45 × 2.3 × 0.65–0.13 g. Note that the arch is somewhat triangular in effective area, so this amounts to about 4 mm in the center and 0 adjacent to the ankles.

Subsequently, the data are re-presented with the amount of thinning on the side of the bridge (Fig. 53).

This was eventually settled on as a more intuitive indicator.

Work Flow on Mass Reduction

One last correlation was carried out. If the luthier elects to reduce foot width, what else does he or she do? The graph presented in Fig. 54 relates foot thickness to arch, center, and top thickness.

The graph shows that top thickness is not related to foot thickness. This should not be surprising. The thickness at the foot and the arch is similar physically, and there is a correlation R^2 of 0.6, which is similar to the correlation between height vs. weight in people. The correlation between foot and center thickness and foot thickness is not as strong; this represents different profiles discussed earlier.

Profile and Failure

The profile that is most common emphasizes prevention of bridge failure. This is a more subtle issue than might first meet the eye. Failures include the following:

- Bridge splitting in two through the center
- Warping
- String damage to the top of the bridge
- Shear failure of the ankles (rotated foot)
- Grain splitting in the thighs
- Broken heart wings
- Broken (kidney) wings
- Broken feet, particularly those that have been very thinned

The thick center of the bridge on the profile would logically counteract bridge splitting. From the side, the bridge looks like a column. It would be more accurate to describe it as a plate. The bending moment and bending moment stresses will be at their maximum in the center. The flexural rigidity, like top plates on a violin, generally increases with thickness cubed. The thicker center would, therefore, counteract the tendency for a bridge to snap in half.

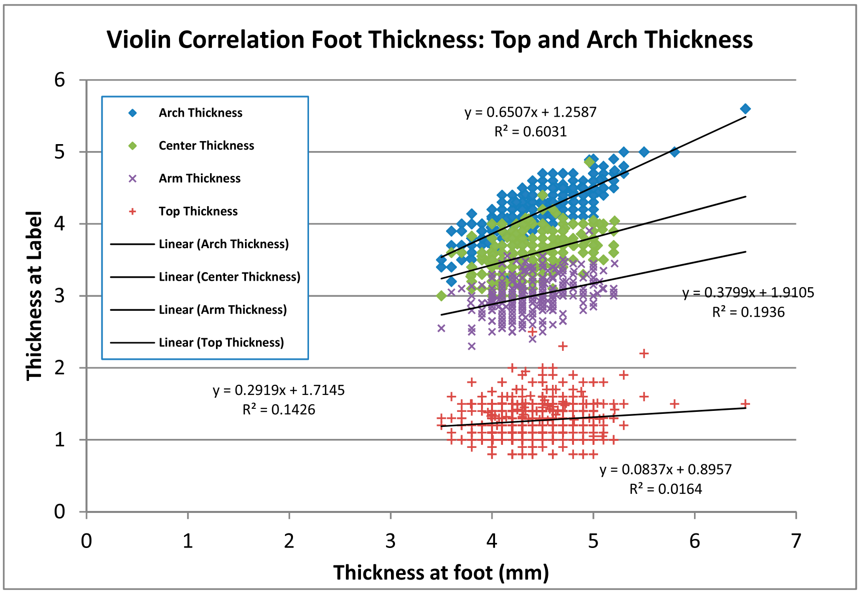


Figure 53. Correlation of foot thickness with thickness at top and at arch.

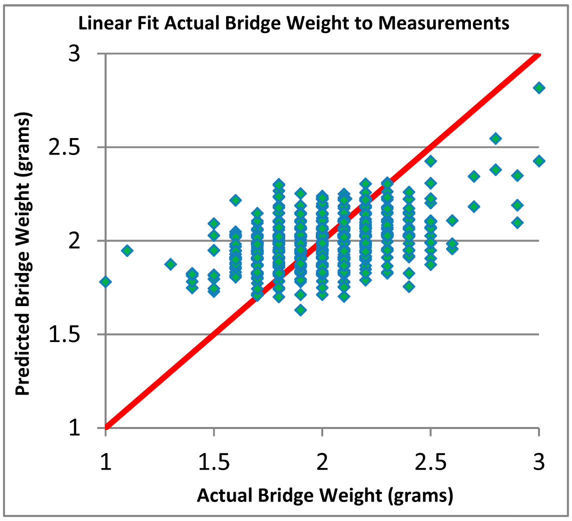


Figure 54. Regression on bridge weight from bridge dimensions.

If the bridge fails with the violin standing on its own, or while being actively played, it indicates the bridge was not strong enough to handle the loads imposed. This is different from if the top of the case is closed on the bridge, which results in it being overloaded. Studies have indicated that people are failure averse, and one would suspect this possibility would be heavily weighted. Restated, any apparent failure will result in steps to correct this possibility. Actually, the bridge splitting in two is vastly preferable to the top being split and the necessity of a sound post job. The underlying question is: If reducing mass on the bridge increases performance, meaning volume and tone, where is the optimum solution? The author has not found a calculation or test of the expected structural limit for a bridge.

String tensions can be obtained from suppliers. D'Addario provides this information for a number of their strings on the Internet. Typical string tensions are shown in the following table:

Violin	Helicore	Zyex
E	18.6	18.6
A	12.7	12.5
D	11.5	10.6
G	10.2	10
	53 lbf	51.7 lbf

String tension is just a bit over 50 pounds force, or about 235 N. The fingerboard-side string angle is about an 8.5° and the tailpiece-side angle of the strings is about 17°. The net downward force calculates out to about 18 lbf downward or 80 N downward force. The cross-sectional area of the center amounts to 0.01597 m × 0.00358 m or 5.717E-5 m^2. This corresponds to about 1.39 MPa on the waist. For the ankles, the stress is higher; for an area of 0.0035 m × 0.00442 m or 1.55E-5 m^2, the stress is about 40 (2 legs)/1.55E-5 or 2.58 MPa. The compressional limit on broadleaf maple, perpendicular to the grain, is about 5.2 MPa for dry wood (12% moisture content). The stress value for sugar maple is 10.1 MPa. For European maple, it will be somewhere in between. The stresses do not seem to be close to this. This does not consider bending stresses. It would seem that some "factor of safety" would be a reasonable expectation. Although a bridge failure is not catastrophic, it is certainly annoying if it occurs with normal use. Therefore, knowing where the minimum bridge thickness is, from a structural perspective, would be helpful.

Warping is a common problem. Most bridges warp in the winter when they dry out from building heat. In general, it appears that the more curvature on the front of the bridge, the more they are inclined to warp. The solution to this is humidification of either the instrument or building. This requires discipline. Building humidification seems to work better and is more uniform.

Linear Correlation

Finally, a linear correlation was made between bridge, foot, ankle, thigh, arm, and waist width as well as foot, thigh, arm center, and top thickness. A simple linear equation was set out with the various measurements multiplied by constant factors. The multipliers were then computed with actual bridge mass data using the differences squared and summed with an Excel What-If Goal Seek analysis. Although this computes a reasonable average, it did not work that well.

Better results were obtained by using a zone-by-zone volume calculation and then using linear multipliers. The zones were picked as follows:

1. Foot thickness × foot length × (0.1 × 0.1 × density × geometrical factor for foot)
2. Ankle width × ankle thickness × (0.1 × 0.1 × density × ankle height × geometrical factor for ankle)
3. Thigh width × arch thickness × bridge width × (0.1 × 0.1 × 0.1 × density × geometrical factor)
4. Waist width × center thickness × (0.1 × 0.1 × density × waist height × geometrical factor)
5. Arm thickness × arm width × (0.1 × 0.1 × density × arm length × geometrical factor)
6. (Center thickness + top thickness) × 0.5 × bridge width × (0.1 × 0.1 × density × area × geometrical factor)

The part in brackets converts millimeter to centimeter and adds some physical dimension to compute volume, which is converted to bridge mass estimate via density. The part in squared brackets is assumed to be sufficiently constant that a reasonable estimate might be derived.

The average error per bridge is about 0.02 g on the second system (Figs. 54 and 55). However, the correlations really do not work that well. As outlined previously, the area cutout is not well represented by thigh and arm width and thickness. Bridge area might be derived from automated photographic interpretation at some time in the future. Bridge height also varies with the fingerboard extension and string height.

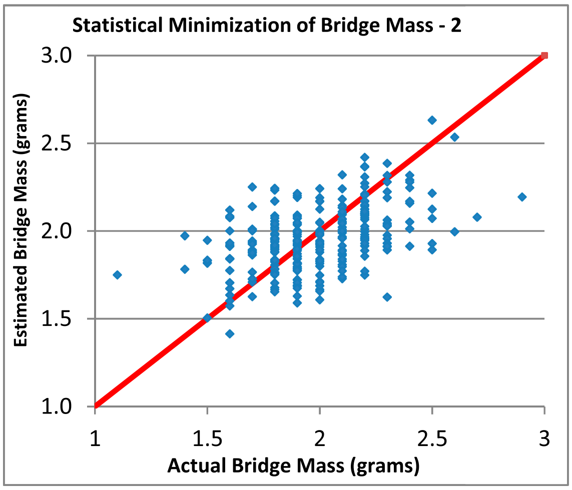


Figure 55. Regression on bridge weight accounting for area weighting and bridge dimensions.

Aesthetic Trimming

One item that the presented statistics do not address is artistic merit. This is of course something that musicians notice. They generally become very attached to their violins. Referring to Johnson and Courtnall (Fig. 56), typical esthetic changes include chamfering on many edges. The esthetic changes reduce weight. The extensive photos in the database help with this evaluation.

One other option would be to include check marks for various trim features in the database. This would be very time-consuming, and it would be difficult to include enough options to cover such variations.

VIOLA

The section on viola has 120 samples. The more complete data, with violin arm thickness and center thickness, comprise some 44 samples. Violas are not standard in size like violins, and the sample is roughly distributed between three groups. Splitting of the groups was carried out on bridge width, as there is an approximate relation with width. Bridges greater than 50 mm were assigned the 420-mm size, bridges between 48 and 50 mm were assigned the 390-mm size, and any bridges less than 48 mm were assigned the smallest size of 390 mm. The breakdown was as follows:

Size	Number
390 mm	104
410 mm	14
430 mm	3

This is not an exact breakdown as the bridge widths follow inexact ranges. Most of the instruments are small violas with some medium sizes and a few large violas.

Mass

Bridge masses for violas are, not surprisingly, heavier than that for violins. The distribution of bridge weights is shown in Fig. 57. Most are more than 2.2 g. and less than 3.18 g.

The average viola bridge weighs 2.94 g, with a standard deviation of 0.33 g (11.2%). The maximum is 3.8 g and the minimum is 2.0 g.

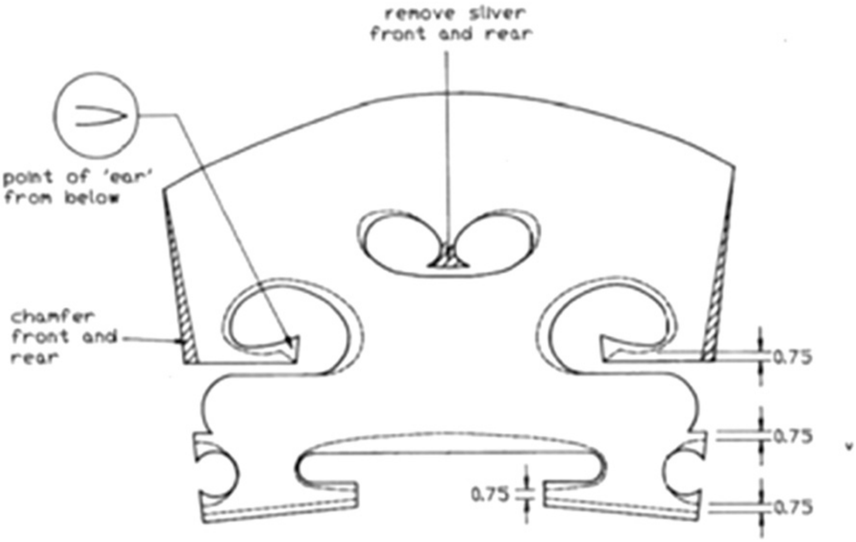


Figure 56. Suggested cutting of the bridge [5].

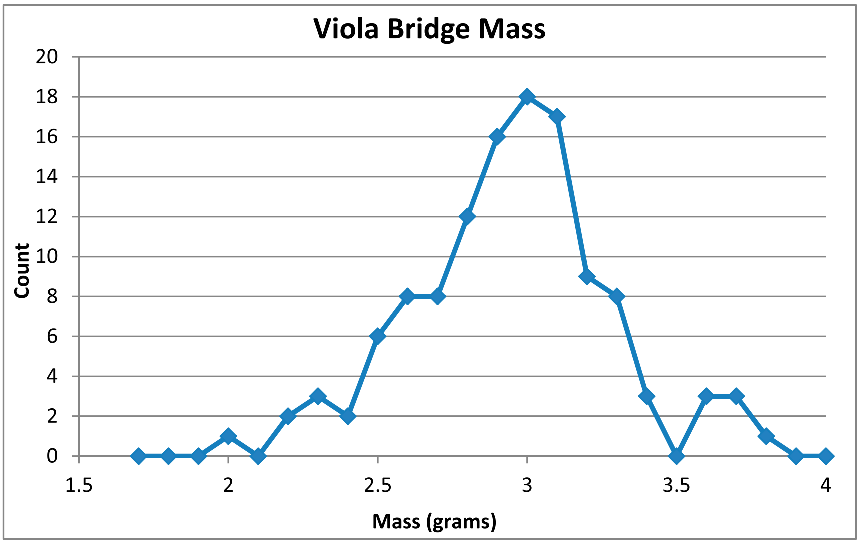


Figure 57. Distribution of bridge mass for viola dataset.

Top Thickness

The average top width is 1.39 mm, with a standard deviation of 0.22 mm (15.8%; Fig. 58). The minimum is 1.0 mm and the maximum is 2.3 mm.

The viola shows no statistical between top thickness and bridge mass (Fig. 59), similar to violins.

Foot Thickness

Foot width is slightly larger than violins, with an average of 13.00 mm for the LHS and 13.05 mm for the RHS (Fig. 60). The standard deviations are 1.03 mm for the LHS and 0.95 mm for the RHS (7.9% and 7.2%;). The maximum is 15.2 mm and the minimum is 7.4 mm.

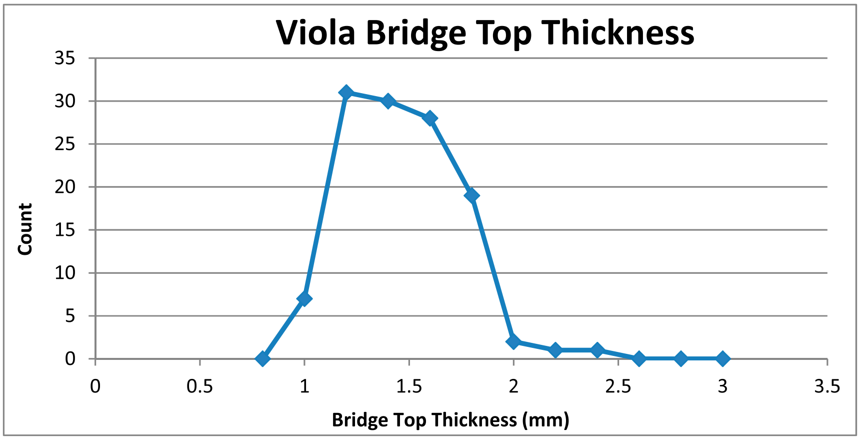


Figure 58. Distribution of bridge top thickness for viola dataset.

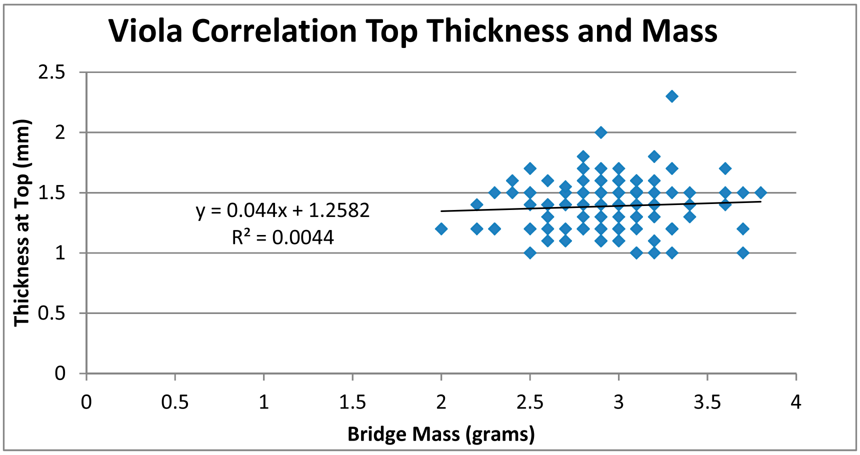


Figure 59. Correlation between viola bridge top thickness and viola bridge mass.

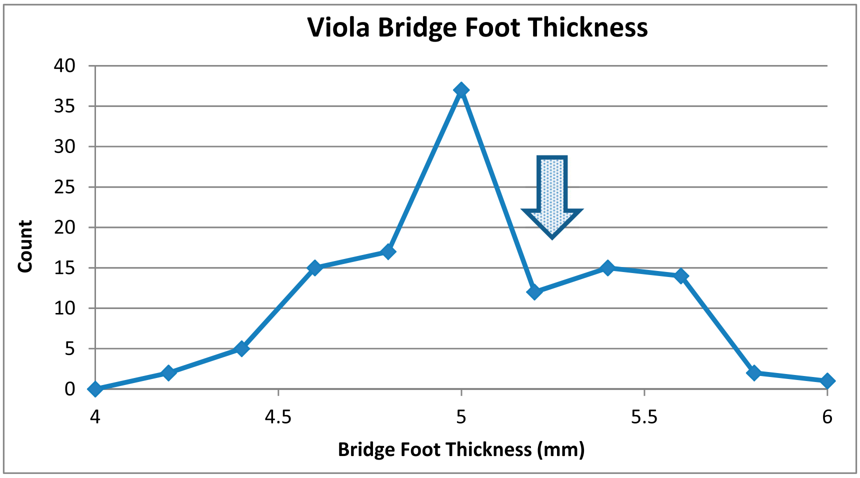


Figure 60. Distribution of foot thickness for viola bridge data set.

Viola Foot Width

The average foot width is 13.00 mm for the LHS and 13.05 mm for the RHS (Fig. 61). Standard deviations are 1.03 mm for the LHS and 0.95 mm for the RHS (7.9% and 7.2%, respectively). The maximum is 15.2 mm and the minimum is 7.4 mm. Foot widths less than 11 mm and more than 14.5 mm are uncommon.

Viola Ankle Width

Ankle widths average 4.03 mm and 4.01 mm for the LHS and RHS, respectively (Fig. 62). The standard deviations were 0.65 mm (16.0%) and 0.66 mm (16.4%) for the LHS and RHS, respectively. The maximum is 5.87 mm and the minimum is 2.5 mm.

Note that there is a hint of a bimodal distribution with a trough in the data shown with an arrow. This will be highlighted in a number of areas. This could be random statistics, a viola size change, or some size standardization on bridge blanks from different manufacturers. This also shows on cellos.

Viola Bridge Thigh Width

Thigh width is shown in Fig. 63. The average thigh widths are 6.0 and 6.0 mm for the left and right, respectively. The standard deviations are

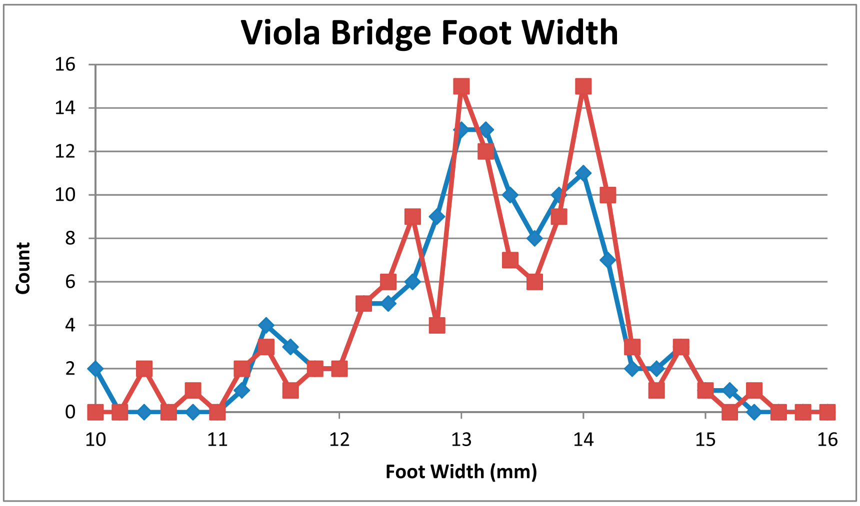


Figure 61. Distribution of foot widths for viola bridge data set.

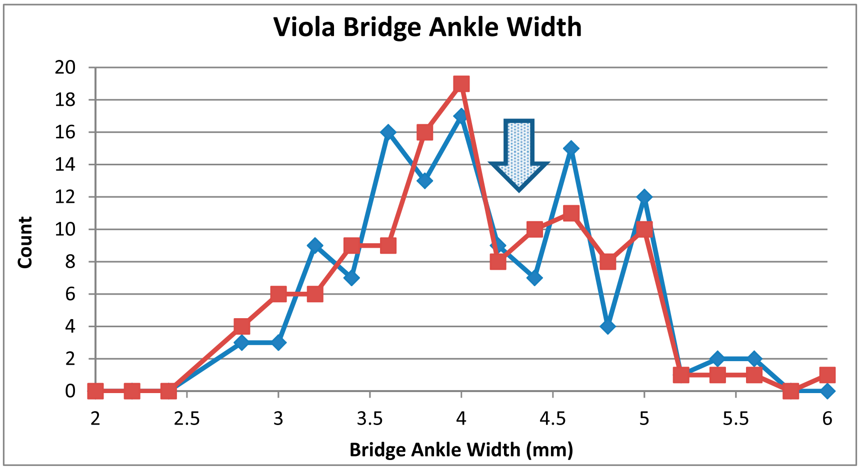


Figure 62. Distribution of bridge ankle widths for viola data set.

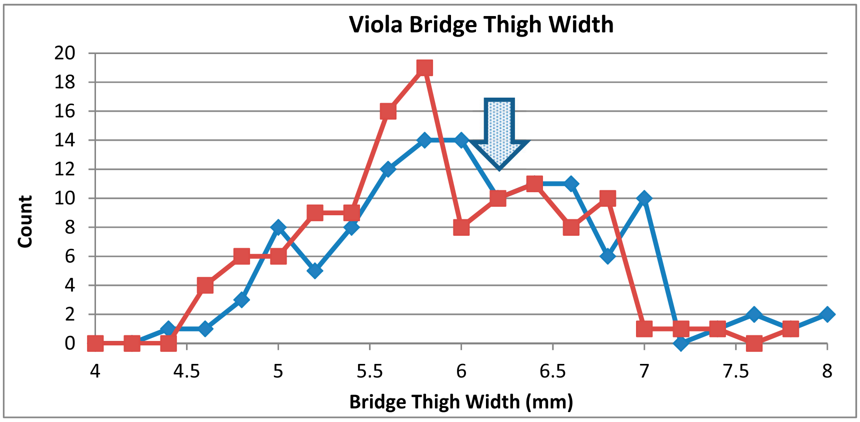


Figure 63. Distribution of bridge thigh widths for viola data set.

0.72 mm and 0.68 mm (12.1% and 11.3%), respectively, for the left and right, with a maximum of 12.1 mm and a minimum of 4.0 mm.

Viola Arm Width

Arm width is more consistent than thighs and ankles, with almost a chapeau (hat) function appearance. The average arm width is 6.42 mm on the LHS and 6.38 mm on the RHS (Fig. 64). The standard deviations are 0.70 mm and 0.67 mm (11.0% and 10.5%) for the LHS and RHS, respectively.

Reviewing the violin distribution also shows somewhat sloped sides, although there are some additional variations associated with a larger statistical sample. The cello distribution is also similar, again with some variations related to a larger sample size.

Viola Arm and Thigh Balance

Data were evaluated to see if luthiers adjusted balance by changing the thickness of the arms and thighs. Probably a 10% (0.9 and 1.1 on graph) variation is just because most of this cutting is done by eye.

There are some data points that are more than 10%. So bridge adjustments may be used to achieve string balance. The cello distribution shows a right hand skew and a second bump at lower waists (Fig. 65).

Viola Waist Width

The waist width is also a bit of an unusual distribution. The violin data also have a slight LHS skew; however, the peak is wider and broader (Fig. 66). This likely reflects a larger statistical sample for the violins. In any event, there are similarities with the violin distribution. The

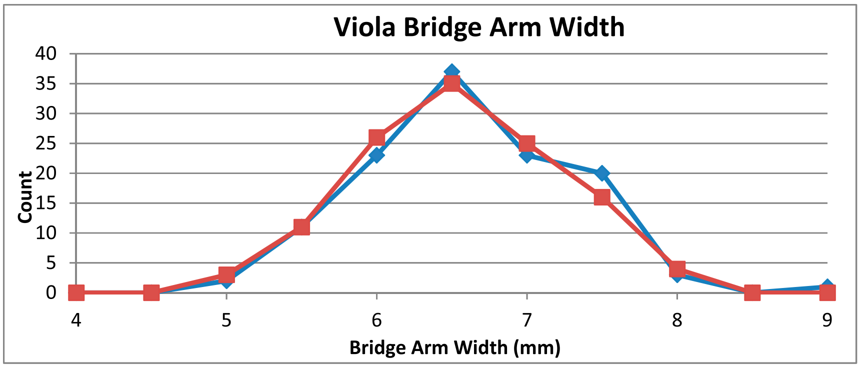


Figure 64. Distribution of bridge arm widths for viola data set.

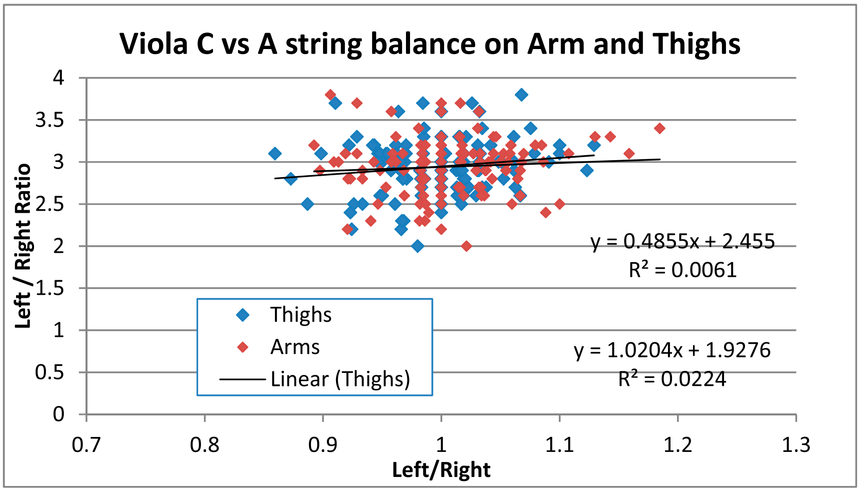


Figure 65. Ratio of arms and thigh widths (C side to A side) for violins (1 = same on both sides).

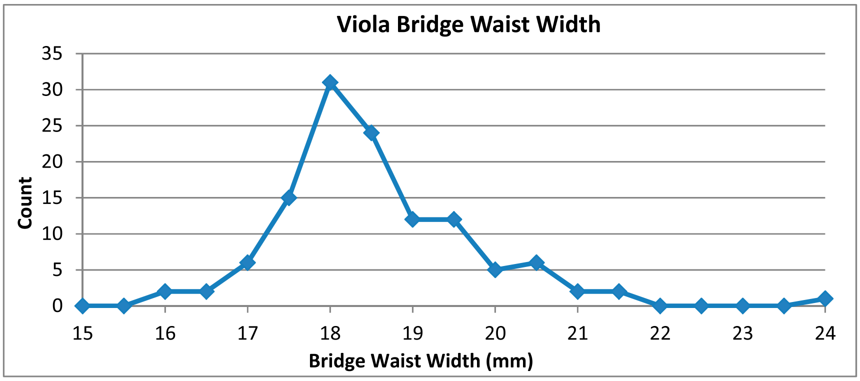


Figure 66. Distribution of bridge waist width for viola data set.

mean waist is 18.37 mm, with widths less than 16 mm as uncommon. The upper limit on common waist sizes appears to be about 21 mm. The standard deviation is 1.18 mm (6.4%), which is less than that for thighs and legs. The maximum is 24 mm and the minimum is 15.85 mm.

Because the waist is in the center of the big X, this would be a change expected to achieve a reduction of Mode 1 (and other modes) frequency. Restated, waist would affect tone.

Viola Bridge Width

Bridge width data are shown in Fig. 67.

The average width is 46.41 mm, with a standard deviation of 1.40 mm (3.0%). The maximum is 50.7 mm and the minimum is 40.7 mm. Recall that bridge width was used to sort viola sizes.

Reducing Mass

As with violins, an attempt was made to see what luthiers might alter if they were trying to reduce mass. There is very little correlation on viola between waist width and mass reduction. A slight reduction trend can be seen (Fig. 68).

An analysis was run to see if the top width correlated with overall bridge weight (Fig. 69). As with the violin, no correlation was observed. This seems to be governed by preventing the strings from cutting into the top of the bridge.

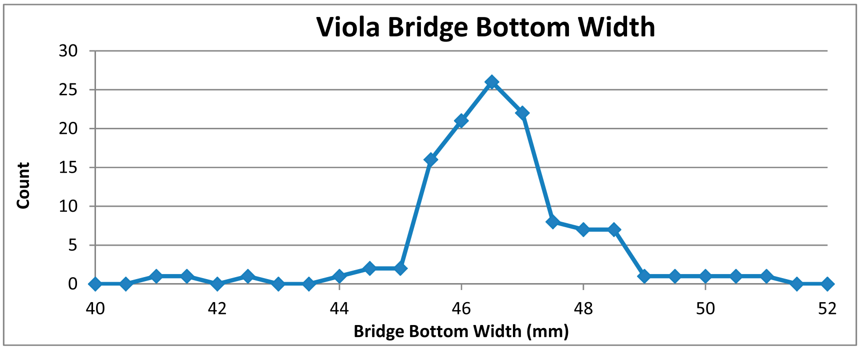


Figure 67. Distribution of bridge bottom width for viola data set.

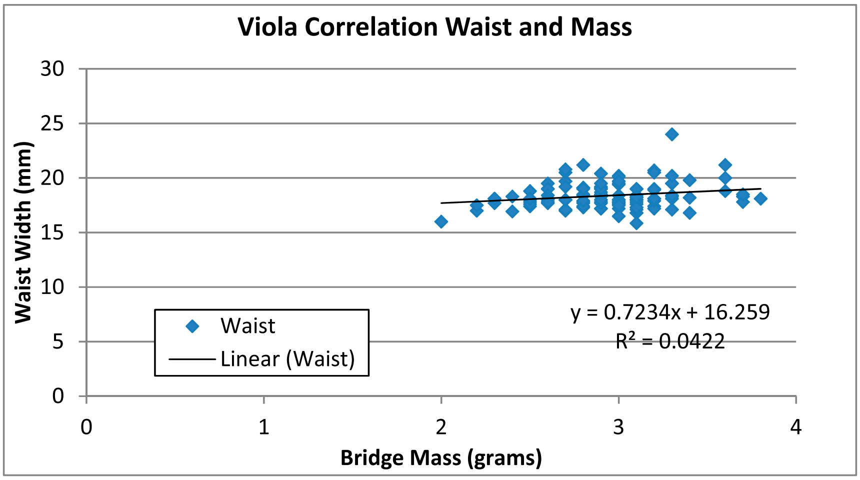


Figure 68. Correlation between bridge waist width and bridge mass viola data set.

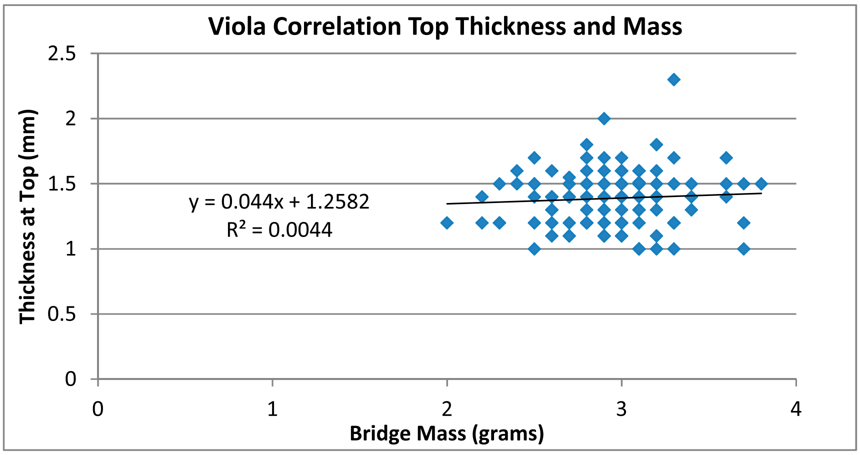


Figure 69. Correlation between bridge top thickness and bridge mass viola data set.

The variation with mass and the thickness of the bridge at various places was also examined (Fig. 70). There is definitely a trend of lower thickness with lower mass. As with the violin, the relation is not really predictive, with the arch thickness having the most predictive power.

A similar analysis was also carried out on thigh width, as shown in Fig. 71.

Any correlation between thigh thickness and mass is a very weak correlation. Thigh thickness is a direct measure of the strength of the bottom limbs of the "big X" and, as discussed previously, may be viewed as more of a change

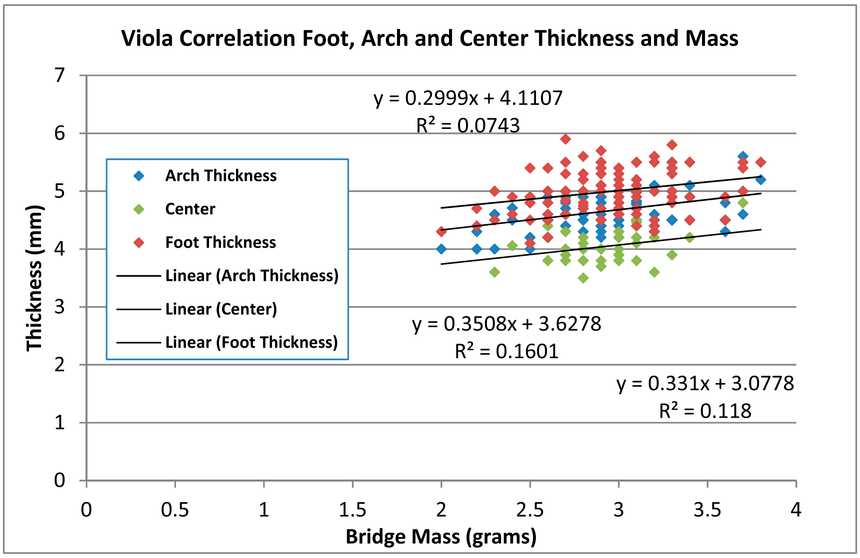


Figure 70. Correlation between foot, arch and center thickness with viola bridge mass.

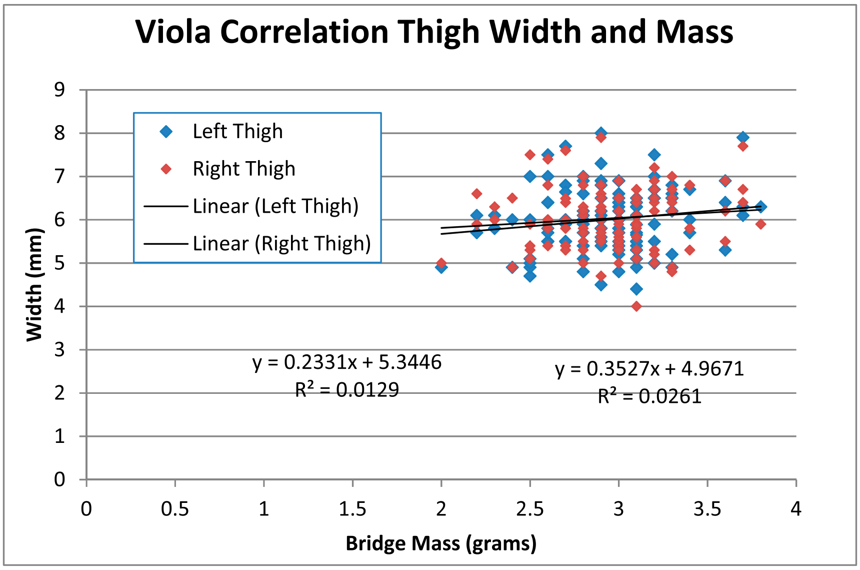


Figure 71. Correlation between thigh width and bridge mass for viola data set.

to affect tone than mass. A similar analysis was carried out on arm width (Fig. 72).

There is an overall trend in these data, which is perhaps a bit steeper than thighs. Again, not much of a correlation is evident. The next analysis was to see if the arch thickness, center thickness, and top thickness were changed systematically with foot thickness (Fig. 73).

There is a correlation between arch thickness and foot thickness. The two are in close physical proximity and the correlation is similar to that of height and weight in people. The center thickness correlation is somewhat weaker. Top thickness and foot thickness are not related.

Profile of Viola Bridges

For the vertical profile, a different technique was used. The data set with center thicknesses is smaller and was sufficiently small that the profile could be presented with all 44 traces. With

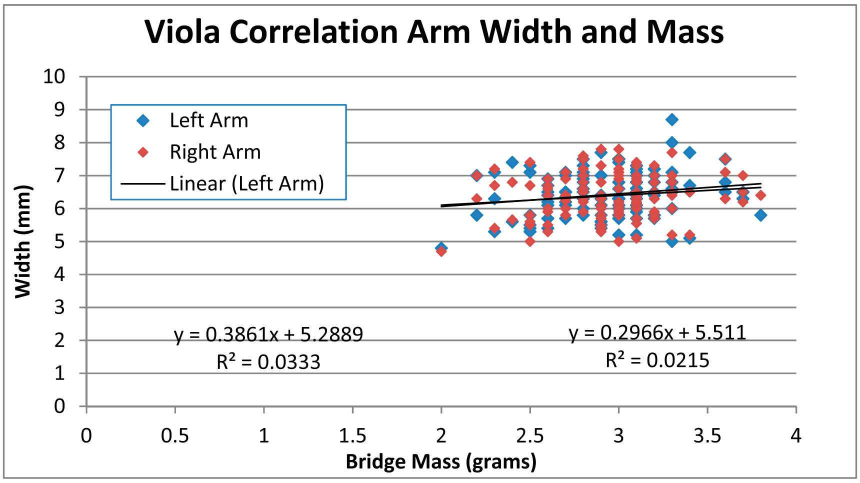


Figure 72. Correlation between arm width and bridge mass for viola data set.

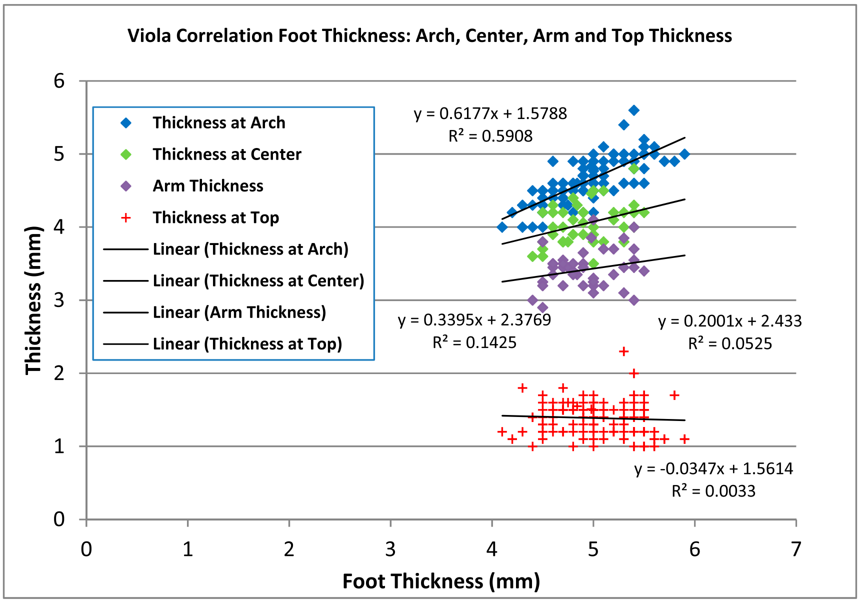


Figure 73. Correlation between arch, center, top and arm thickness and foot thickness viola data set.

the larger data sets for cellos and violins, this becomes a mess without averaging.

The red curve at the top shows feet that are thinner than the arch (Fig. 74). Note that there is a break over at about 20 mm. Note that the bridge distance from the foot was estimated off a blank, and the real data would vary from bridge to bridge. These data are not recorded and the aforementioned should be viewed as a reasonable approximation, not absolute data. Note also that arm thickness is affected by side-to-side or transverse thinning, and the distance has been adjusted a bit to smooth the profile, consistent with general observation. This is a natural transition into lateral thinning. An indicator has been used for this. The indicator has been changed slightly from the violin data. In this case, the data have been plotted as arm thickness divided by center thickness, instead of center thickness divided by arm thickness (Fig. 75). It is the same data, just a slightly different presentation.

The data show that, on average, the arms are 0.84 of the center thickness or 16% thinner. The standard deviation on the 0.84 is 0.04. The maximum thinning corresponds to 0.71 or a 29% reduction, and the minimum thinning correlates to a ratio of 0.93 or a 7% thinning of the arms. There are only a few bridges that do not have much lateral thinning.

CELLO

The cello data set has 172 samples. The more complete data, with violin arm thickness and center thickness, comprise some 57 samples. There were some fractional sized cellos in the data set. They were deleted from the material in

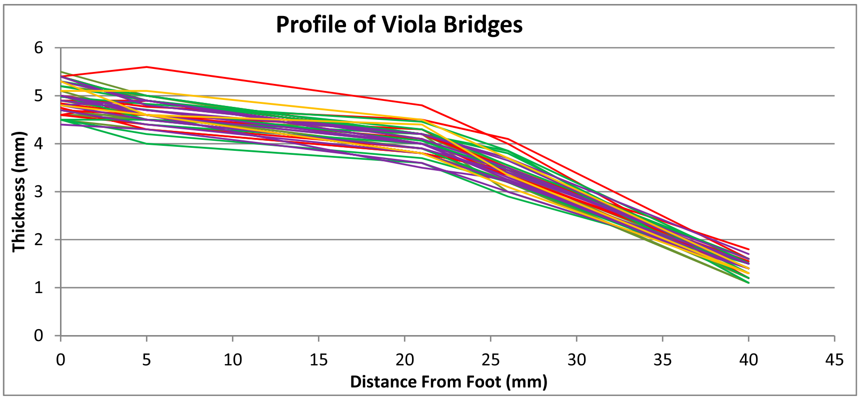


Figure 74. Profile of vertical thickness for voila bridges.

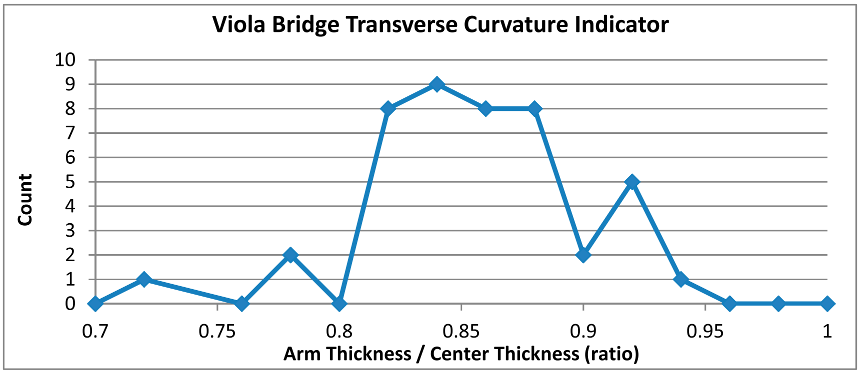


Figure 75. Transverse curvature indicator arm thickness as fraction of center thickness viola data set.

this report. There were not enough of the various fractional sizes to provide statistically valid data, at least not at this point in time.

Strobel, in Useful Measurements for Violin Makers, suggests that most measurements decrease by approximately 8% for each size reduction below 4/4 in the series 3/4, 1/2, 1/4, 1/8, 1/16, and 1/32. The approximate is heavily emphasized. For a half-sized instrument, this means 0.92 × 0.92 = 0.85. This means multiplying the dimension by 0.85, or reducing it by 15%.

There is another important variation with cellos: two standard geometries of bridges. For the more complete data set, that includes arm and center thickness, the bridges were split by viewing the bridge photos. The results are as follows:

Bridge	Number	Percent
French	44	79
Belgian	12	21
Total	56	100

Overall, a reasonable estimate is that 80% of bridges are of French design and 20% are of Belgian design. Considerable discussion of vibration modes was contained in earlier parts of the article.

Mass

Mass data for the cello bridges is presented in Fig. 76. The average mass is 15.3 g, with a standard deviation of 1.8 g (12%). The lightest was 11.6 g and the heaviest was 20.8 g. The data suggest weights under 12 g are uncommon and weights over 20 g are very rare. Interestingly, the average weight of the 44 French bridges was 15.10 g and the 12 Belgian bridges was 15.05 g. This is a very small difference in average mass.

Cello Top Thickness

As with the previous instruments, the thickness of the top was plotted up (Fig. 77). The lowest data, that is, at 1.5 mm, showed signs of damage in the photograph, where the strings had dug into the top of the bridge. The average top thickness is 2.4 mm, with a standard deviation of 0.4 mm (17.5%) and a minimum of 1.5 mm and a maximum of 4.5 mm. Most thicknesses are higher than 1.8 mm and less than 3.2 mm.

Cello Foot Thickness

Cello foot thickness averaged 11.3 mm, with a standard deviation of 1.8 mm (12.0%; Fig. 78). The maximum is 20.8 mm and the minimum is 8.8 mm.

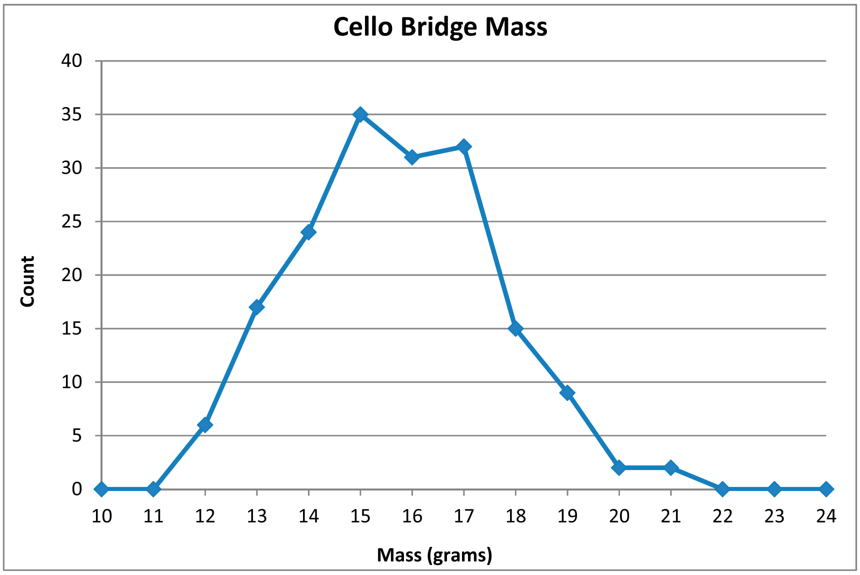


Figure 76. Distribution of bridge mass for cello data set.

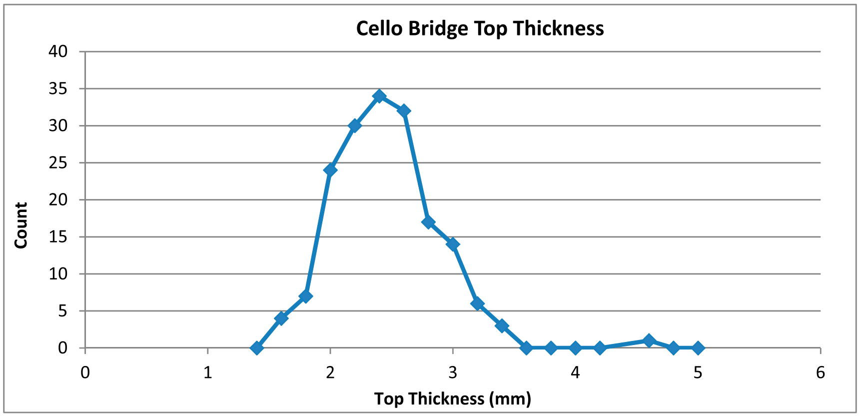


Figure 77. Distribution of bridge top thickness for cello data set.

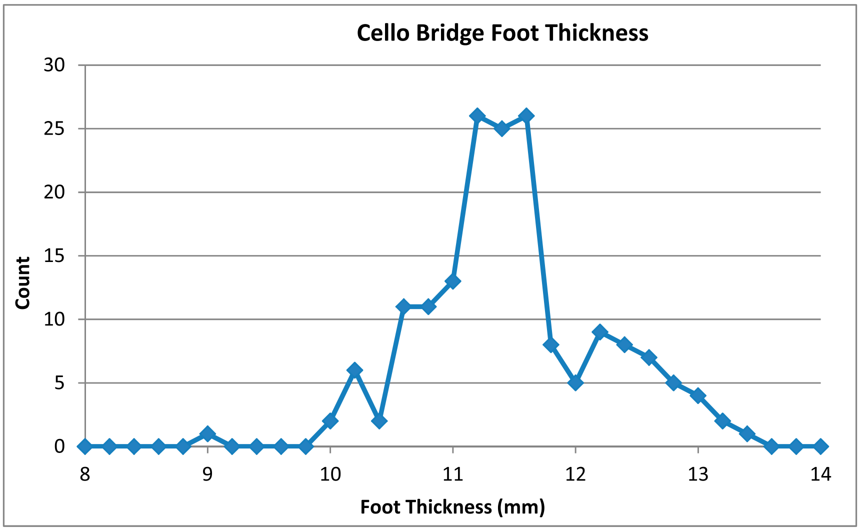


Figure 78. Distribution of foot thickness for cello bridge data set.

Cello Bridge Feet Width

The average foot width is 23.7 mm for the LHS and 23.7 mm for the RHS (Fig. 79). Standard deviations are 2.5 mm for the LHS and 2.7 mm for the RHS (10.6% and 11.5%, respectively). This is more percent variation than for violins and violas. The maximum is 30.2 mm and the minimum is 11.8 mm. Foot widths less than 21 mm and more than 27 mm are uncommon.

Cello Bridge Ankle Width

For the cello, the ankle would be better described as lower leg width. The average ankle width is 8.4 mm and 8.7 mm for the LHS and RHS, respectively (Fig. 80). The standard deviations are 0.9 mm (8.2%) and 0.8 mm (8.8%) for the LHS and RHS, respectively. The maximum is 10.5 mm and the minimum is 6.4 mm.

Cello Bridge Thigh Width

The thigh width is shown in Fig. 81. The average thigh widths are 10.9 and 10.9 mm for the left and right, respectively. The standard deviations are 0.9 mm and 0.9 mm (8.2% and 8.0%), respectively, with a maximum of 13.4 mm and a minimum of 8.0 mm. The shaded blue arrow may mark different blank manufacturers.

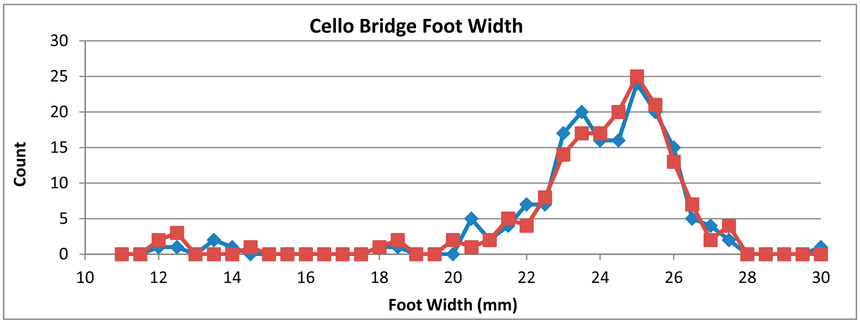


Figure 79. Distribution of foot widths for cello bridge data set.

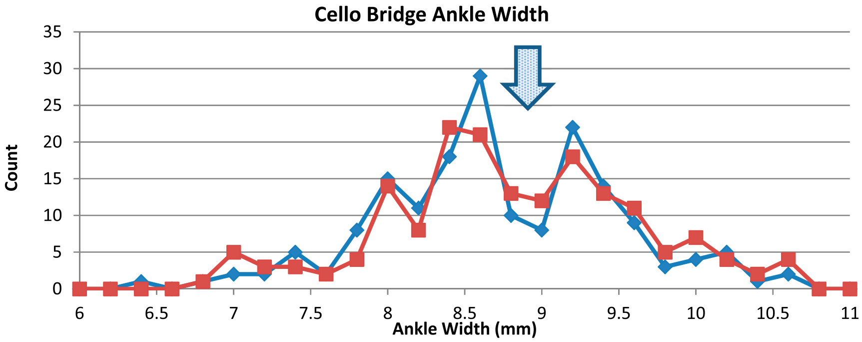


Figure 80. Distribution of bridge ankle widths for cello data set.

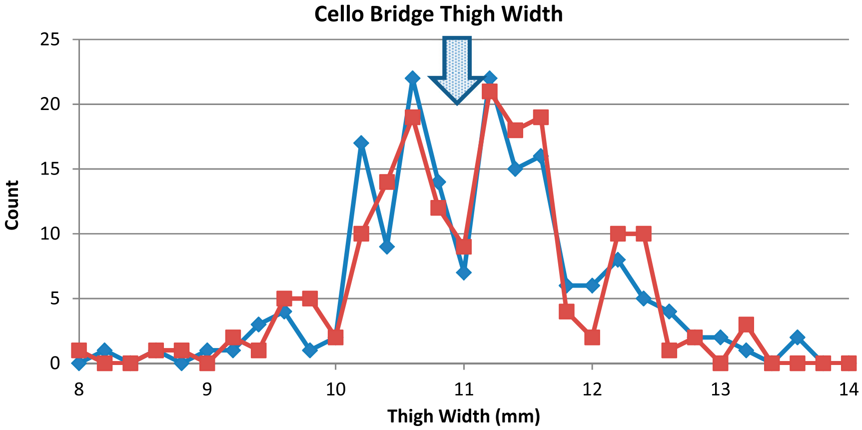


Figure 81. Distribution of bridge thigh widths for cello data set.

Cello Arm Width

The average arm width is 9.8 mm on the LHS and 9.9 mm on the RHS (Fig. 82). The standard deviations are 1.3 mm and 1.3 mm (13.4% and 13.4%), respectively, for the LHS and RHS. Distribution is similar in violins and violas.

Cello Arm and Thigh Balance

Data were evaluated (Fig. 83) to see if luthiers adjusted balance by changing the thickness of the arms and thighs. Probably, a 10% (0.9 and 1.1 on graph) variation is just because most of this cutting is done by eye.

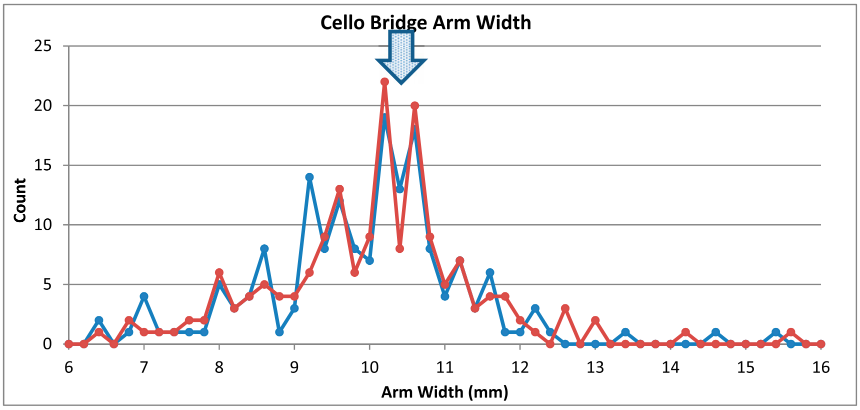


Figure 82. Distribution of bridge arm widths for cello data set.

There are some data points that are over 10%. So bridge adjustments may be used to achieve string balance. Note there is a dense line of points right in the middle on 1.0.

Cello Waist Width

The waist width is also a bit of an unusual distribution (Fig. 84). The violin and viola data also have a slight LHS skew. For cellos, like violins, the peak is wider and broader. The mean waist is 31.4 mm, with widths less than 30 mm as uncommon. The upper limit on common waist sizes appears to be about 36 mm. The standard deviation is 2.1 mm (6.7%), which is less than that for thighs and legs. The maximum is 36.0 mm and the minimum is 25.9 mm.

Because the waist is in the center of the big X, this would be a change expected to achieve a reduction of Mode 1 (and other modes) frequency. Center thickness would also be important in this area.

Cello Bottom Bridge Width

Bridge width data is shown in Fig. 85.

The average width is 89.9 mm, with a standard deviation of 2.5 mm (2.8%). The maximum is 102.5 mm and the minimum 83.0 mm. Recall that bridge width was used to sort cello sizes and in particular to identify fractional sizes (which were taken out of the sample set).

Reducing Mass

As with violins, an attempt was made to see what luthiers might alter if they were trying to reduce mass. There is very little correlation on violin and viola between waist width and mass

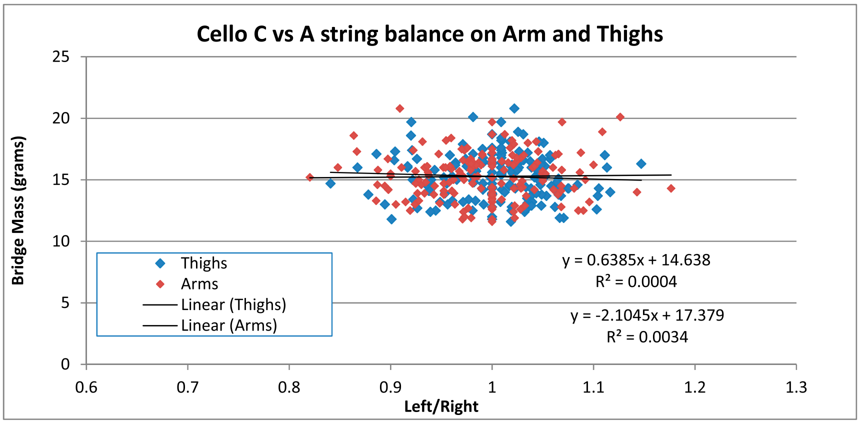


Figure 83. Ratio of arms and thigh widths (C side to A side) for violins (1 = same on both sides).

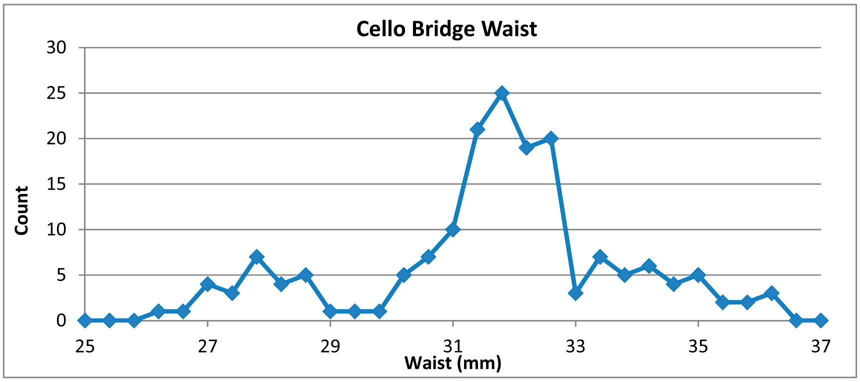


Figure 84. Distribution of bridge waist width for cello data set.

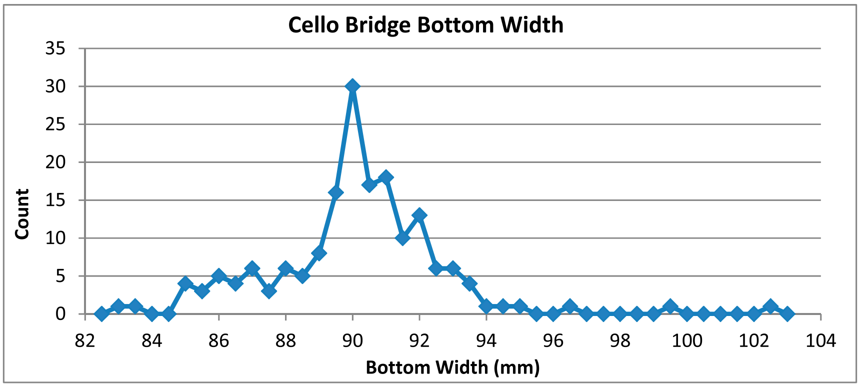


Figure 85. Distribution of bridge bottom width for cello data set.

reduction. A slight reduction trend can be seen on violin and viola.

The variation with mass and the thickness of the bridge at various places was also examined (Fig. 86). Note that for the cello, a separate plot was not made for top thickness, and it was added to the other data. There is definitely a trend of lower thickness with lower mass. As with the violin, the relation is not really predictive, with the arch thickness having the most predictive power. As with previous top thickness analysis, there was no correlation between bridge top thickness and bridge mass (Fig. 87).

A similar analysis was also carried out on thigh width, as shown in Fig. 88.

There is no correlation between thigh thickness and mass. Thigh thickness is a direct measure of the strength of the bottom of the "big X" and, as discussed previously, may be viewed as more of a change to affect tone than mass. A similar analysis was carried out on arm width (Fig. 89).

Again, not much of a correlation is evident. The next analysis was to see if the arch thickness, center thickness, and thickness at the top were changed systematically with foot thickness (Fig. 90).

For the violin and viola, there is a correlation of about 0.6 between arch thickness and foot thickness. The two are in close physical proximity and the correlation is similar to that of height and weight in people. For the cello, there is no physical proximity and the relation becomes much weaker with an R^2 of 0.22. There is a general downward trend for arch, center, and arm thickness. There is a bit of slope with the top thickness. However, this is weak. Other instruments were weaker on this correlation.

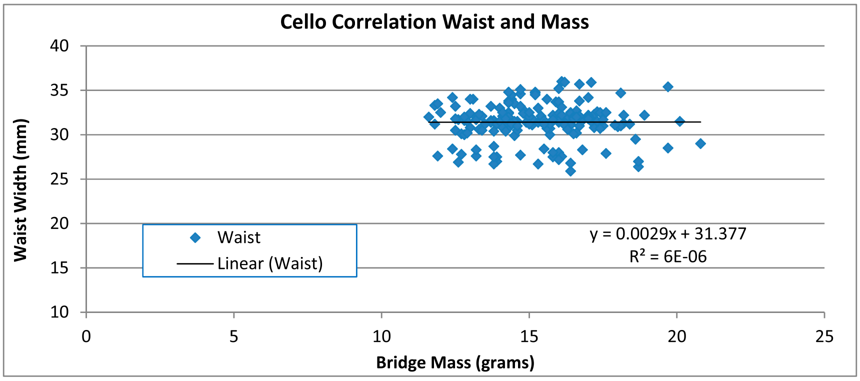


Figure 86. Correlation between waist width and bridge mass cello data set.

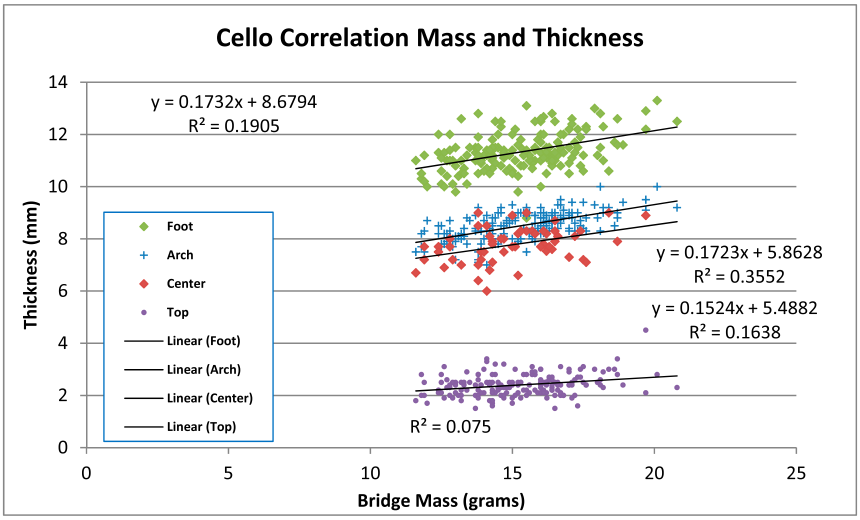


Figure 87. Correlation between foot, arch, center and top thickness with cello bridge mass.

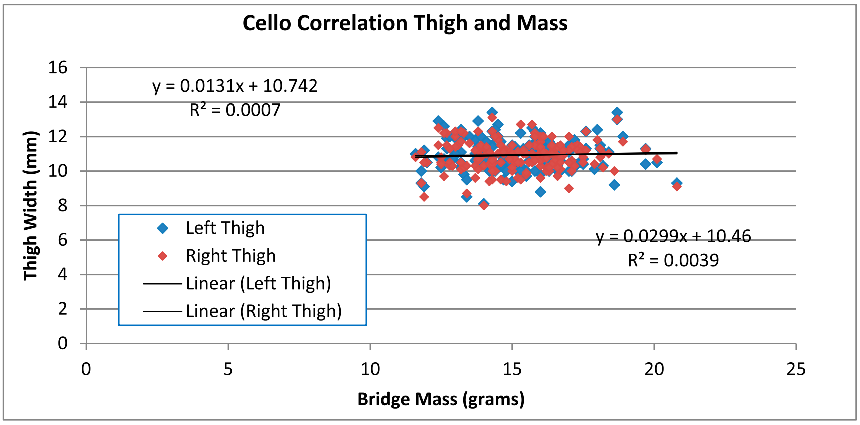


Figure 88. Correlation between thigh width and bridge mass for cello data set.

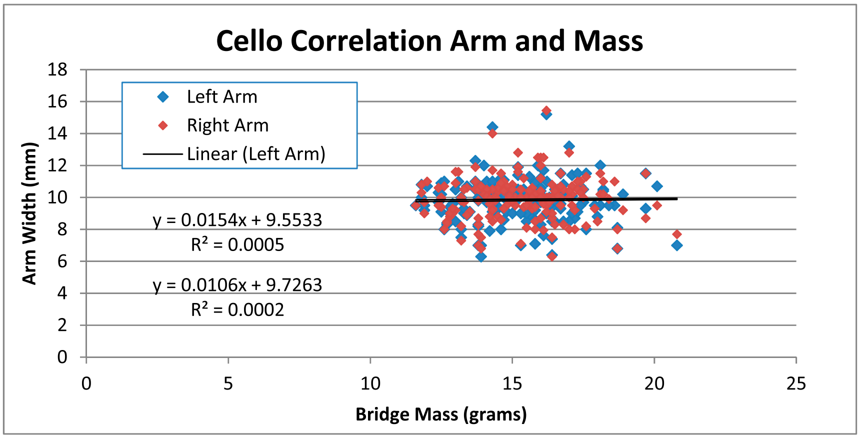


Figure 89. Correlation between arm width and bridge mass for cello data set.

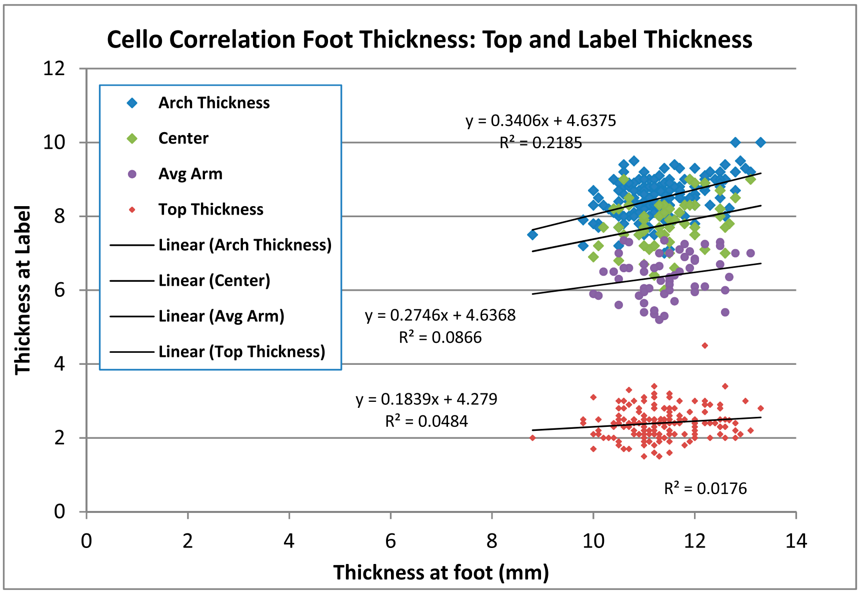


Figure 90. Correlation between arch, center, top and arm thickness with foot thickness cello data set.

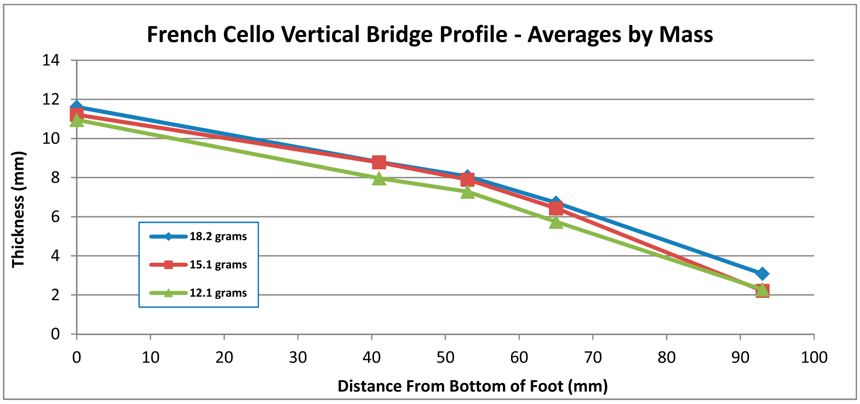


Figure 91. Profile of vertical thickness for French cello bridges.

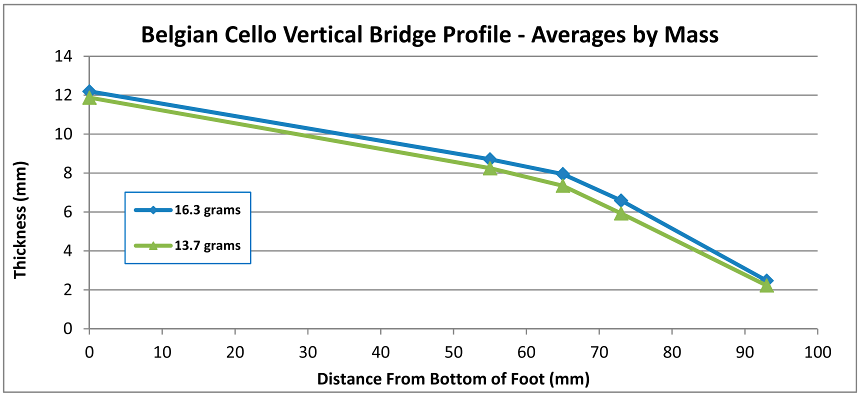


Figure 92. Profile of vertical thickness for Belgian cello bridges.

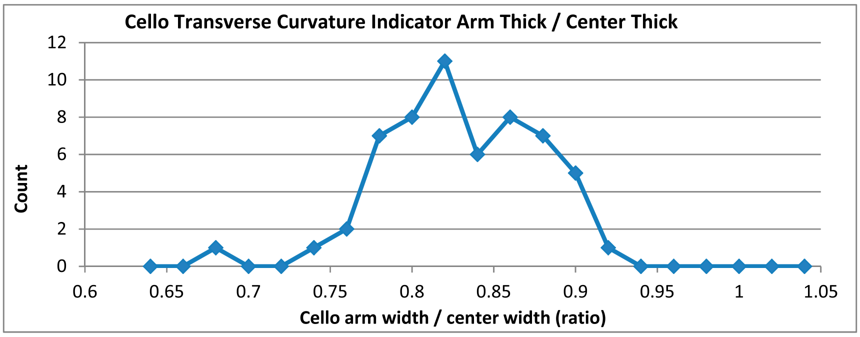


Figure 93. Transverse curvature indicator arm thickness as fraction of center thickness cello data set.

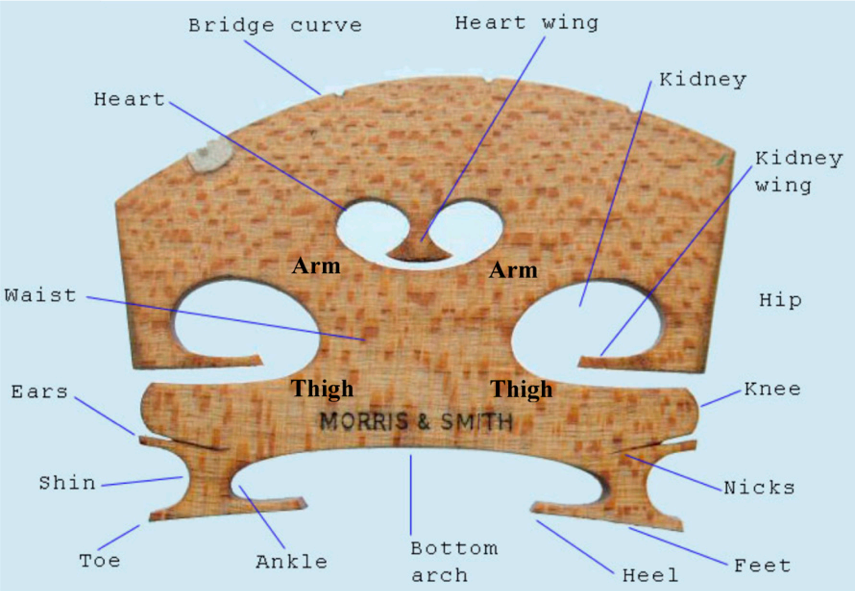


Figure 94. Bridge terminology.

Table 1. Summary table of statistics for all bridges.

	Width	Width	Width	Width	Width	Width	Width	Width	Width	Width	Thickness	Thickness	Thickness	Mass	Thickness	Thickness	Thickness	Arm/ Center Ratio
	Bottom	L Foot	L Ankle	L Thigh	L Arm	Waist	R Arm	R Thigh	R Ankle	R Foot	Feet	Arch	Top		L Arm	R Arm	Center	
Data	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	g	mm	mm	mm	
										Violin								
Count	764	764	766	765	764	765	764	765	765	764	765	766	766	766	310	310	305	303
Average	40.80	11.53	3.51	5.50	5.27	15.97	5.26	5.54	3.50	11.58	4.42	4.14	1.27	2.00	2.94	3.04	3.58	0.8
Standard Dev.	0.99	0.80	0.64	0.55	0.53	1.12	0.53	0.57	0.63	0.76	0.34	0.29	0.22	0.26	0.27	0.25	0.27	0.05
SD percent	2.4	6.9	18.2	10.0	10.1	7.0	10.0	10.4	18.0	6.6	7.7	6.9	17.7	13.0	9.3	8.2	7.7	5.5
Maximum	45.22	14.2	7	7.3	7.3	21	7.42	7.2	6.6	14.2	6.5	5.6	2.5	3	3.9	3.91	4.86	0.98
Minimum	34	8	1.8	3.1	3.2	11.71	3.5	2.5	2	8.8	3.5	3	0.8	0.8	2.2	2.4	3	0.65
										Viola								
Count	120	120	120	120	120	120	120	120	120	120	120	120	119	120	44	44	44	44
Average	46.41	13.00	4.03	6.00	6.42	18.37	6.38	6.02	4.01	13.05	4.99	4.66	1.39	2.94	3.35	3.49	4.05	0.84
Standard Dev.	1.40	1.03	0.65	0.72	0.70	1.18	0.67	0.68	0.66	0.95	0.36	0.29	0.22	0.33	0.28	0.27	0.27	0.04
SD percent	3.0	7.9	16.0	12.1	11.0	6.4	10.5	11.3	16.4	7.2	7.3	6.3	15.8	11.2	8.5	7.7	6.6	4.9
Maximum	50.7	15	5.6	8	8.7	24	7.8	7.9	5.87	15.2	5.9	5.6	2.3	3.8	4.2	4	4.8	0.93
Minimum	40.7	7.4	2.5	4.4	4.8	15.85	4.7	4	2.5	10.3	4.1	4	1	2	2.8	3	3.5	0.71
										Cello								
Count	172	172	172	172	171	172	171	172	172	172	172	172	172	173	59	59	57	57
Average	89.9	23.7	8.6	10.9	9.8	31.4	9.9	10.9	8.7	23.7	11.3	8.5	2.4	15.3	6.4	6.5	7.8	0.8
Standard Dev.	2.5	2.5	0.7	0.9	1.3	2.1	1.3	0.9	0.8	2.7	0.7	0.5	0.4	1.8	0.6	0.6	0.7	0.0
SD percent	2.8	10.6	8.4	8.2	13.4	6.7	13.4	8.0	8.8	11.5	6.5	6.3	17.5	12.0	9.9	9.7	8.6	5.7
Maximum	102.5	30	10.5	13.4	15.2	36	15.44	13.1	10.5	30.2	13.3	10	4.5	20.8	7.6	7.8	9	0.92
Minimum	83	12	6.4	8.1	6.3	25.9	6.3	8	6.8	11.8	8.8	7	1.5	11.6	5	5.2	6	0.68

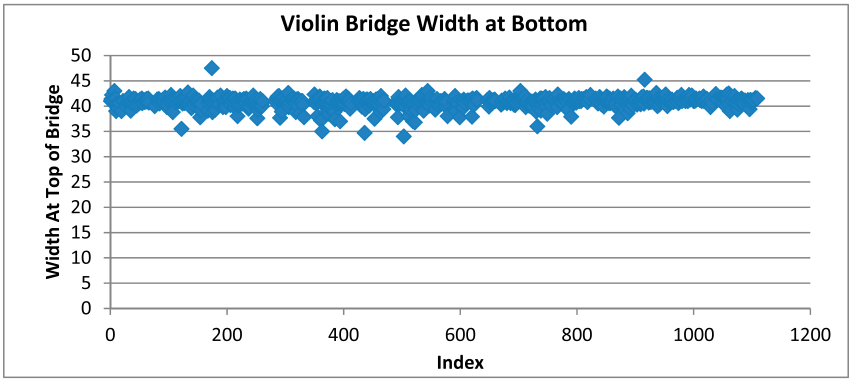


Figure 95. Raw data violin bridge width at bottom (foot).

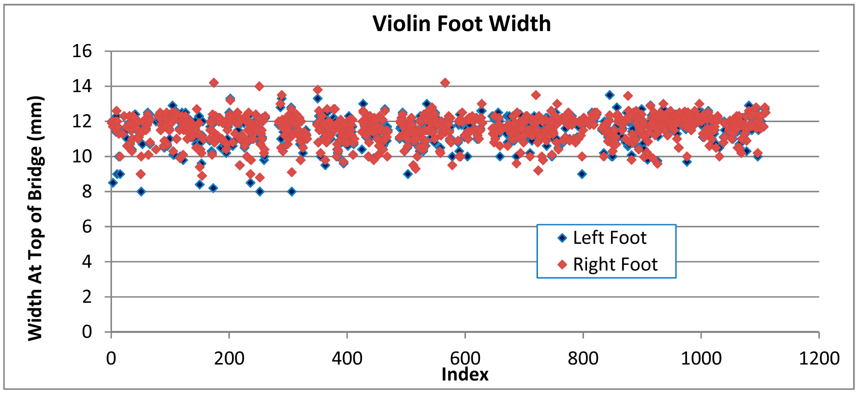


Figure 96. Raw data violin foot width.

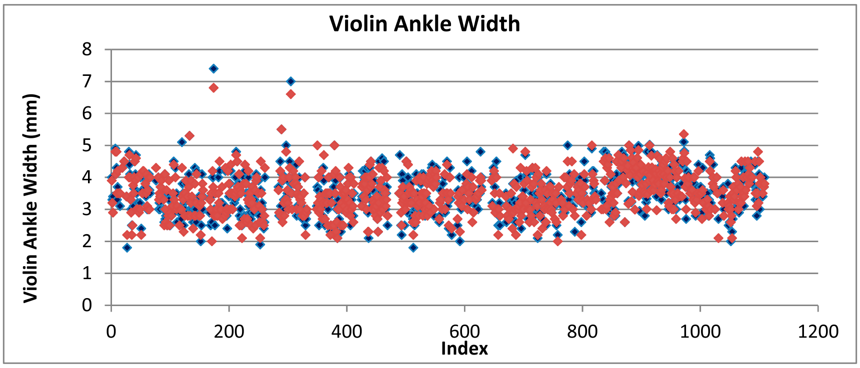


Figure 97. Raw data violin ankle width.

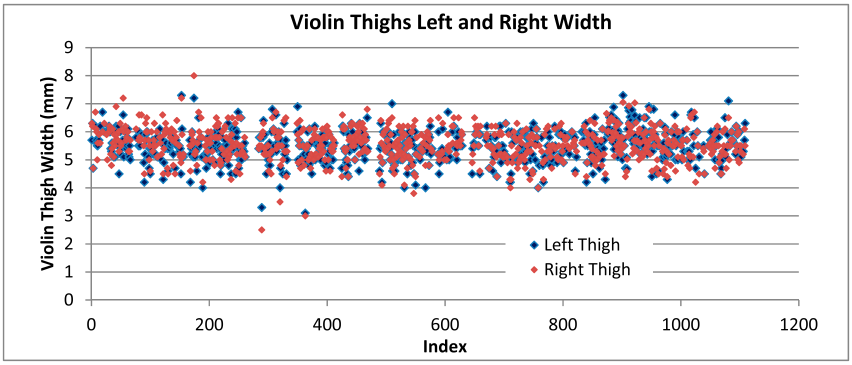


Figure 98. Raw data violin thighs left and right.

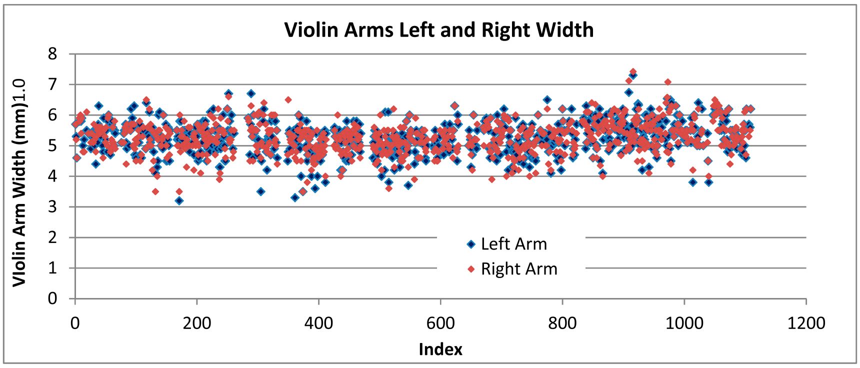


Figure 99. Raw data violin arm widths left and right.

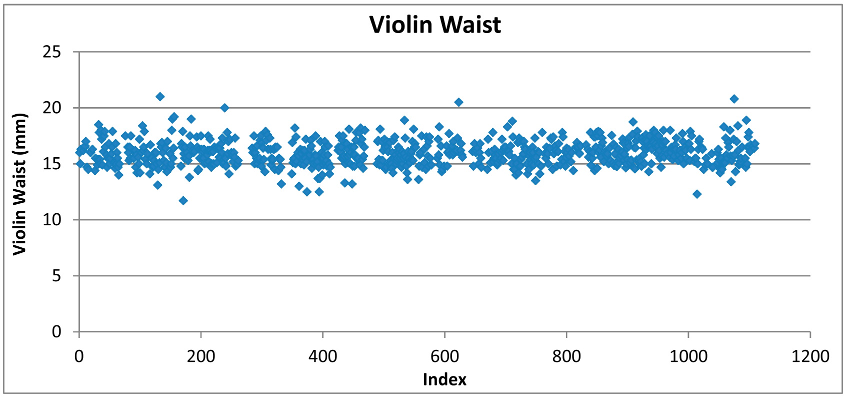


Figure 100. Raw data violin waist width.

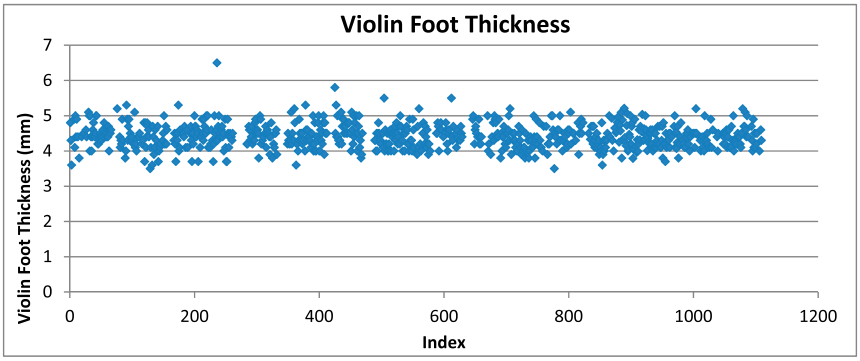


Figure 101. Raw data violin foot thickness.

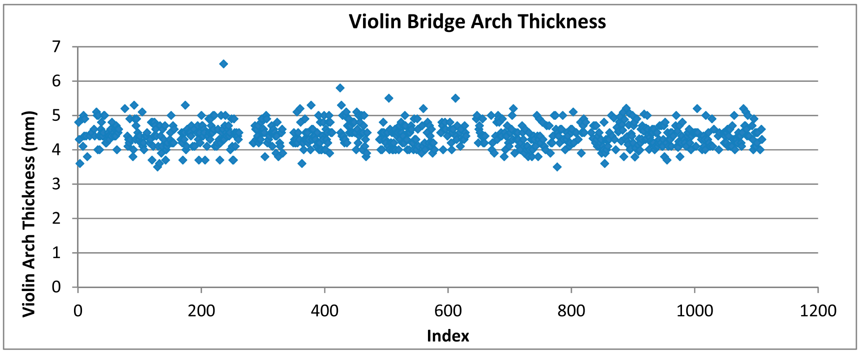


Figure 102. Raw data bridge arch thickness.

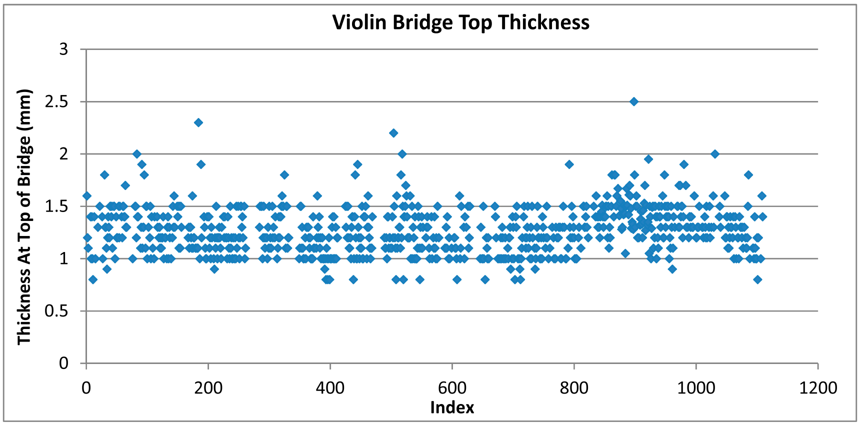


Figure 103. Raw data bridge top thickness.

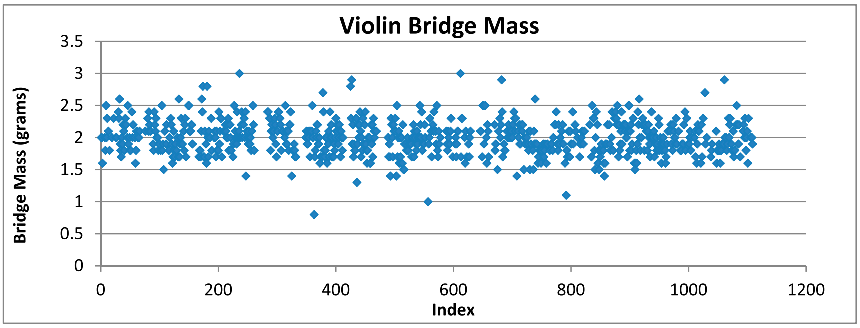


Figure 104. Raw data bridge mass violins.

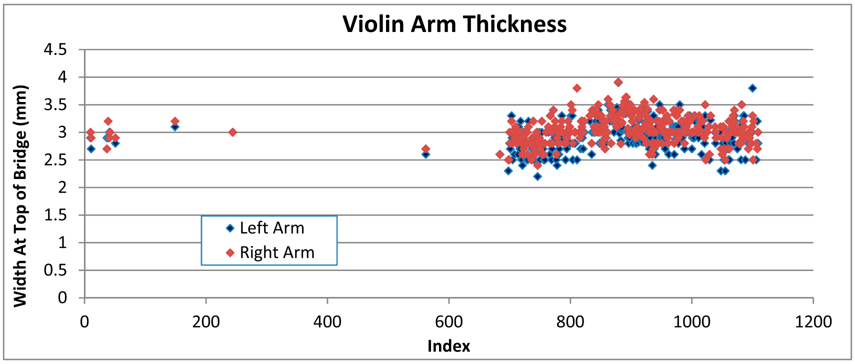


Figure 105. Raw data bridge arm thickness.

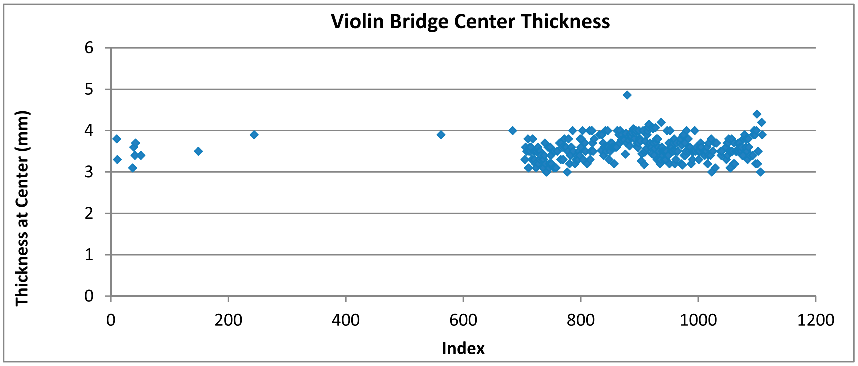


Figure 106. Raw data bridge center thickness.

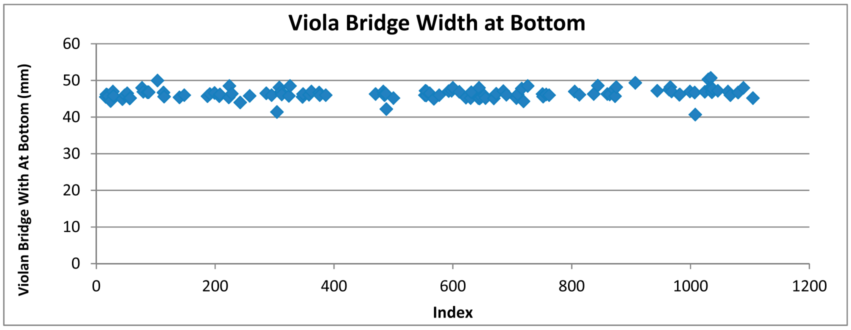


Figure 107. Raw data viola bridge width at bottom (foot).

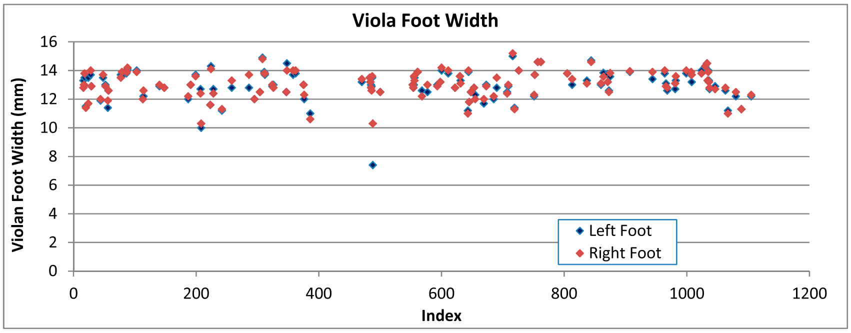


Figure 108. Raw data viola foot width.

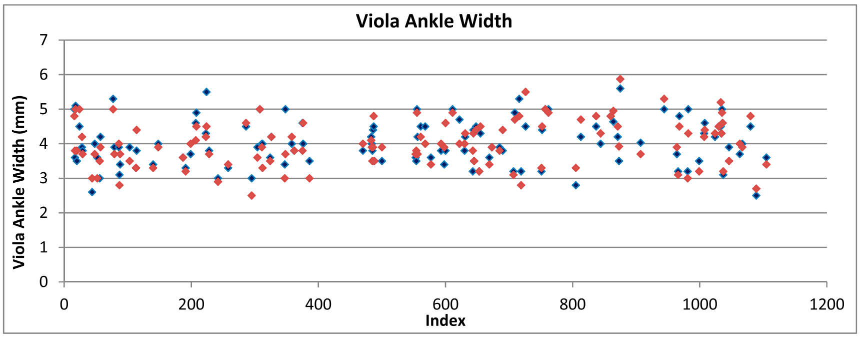


Figure 109. Raw data viola ankle width.

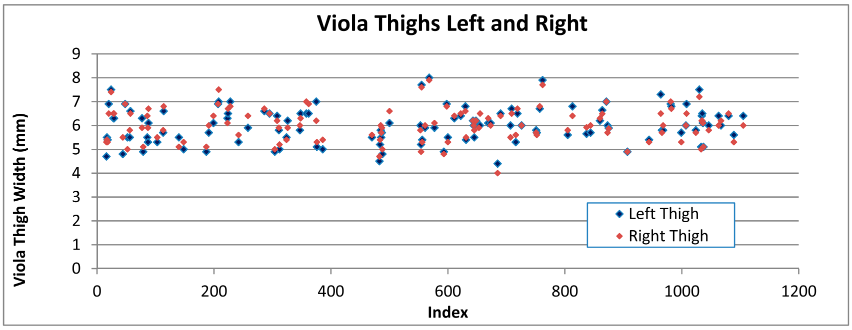


Figure 110. Raw data viola thighs left and right.

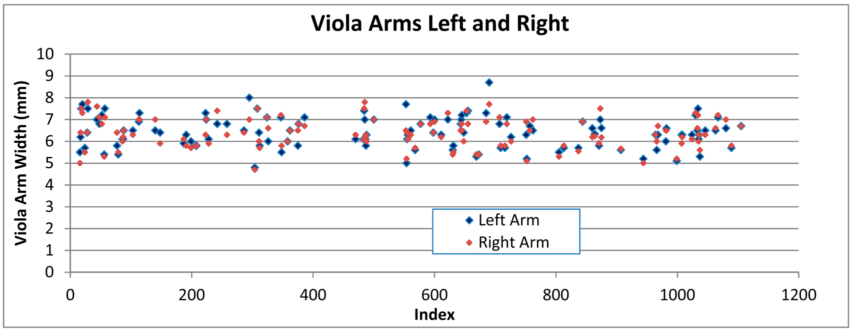


Figure 111. Raw data viola arm widths left and right.

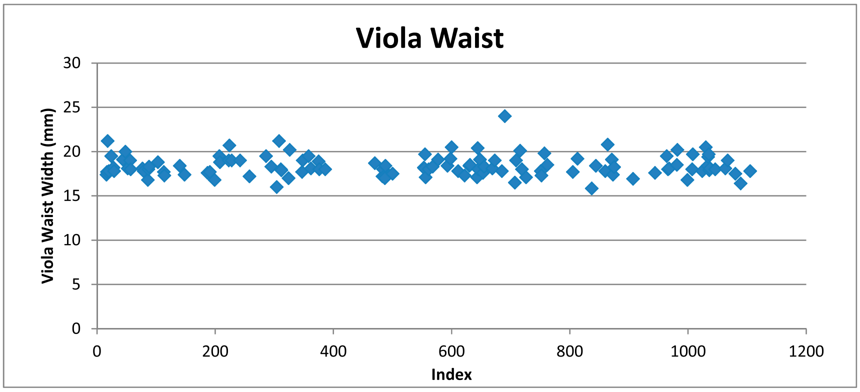


Figure 112. Raw data viola waist width.

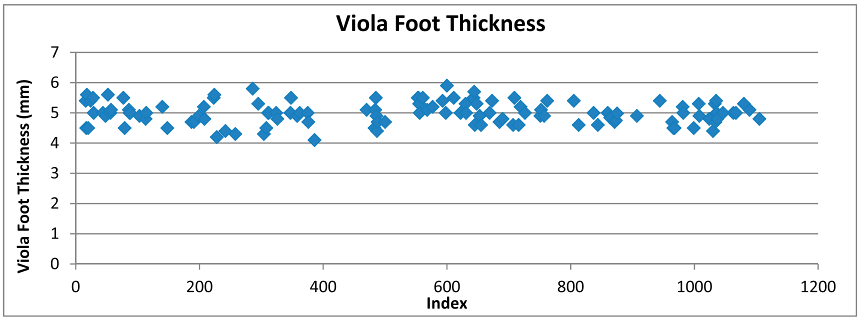


Figure 113. Raw data viola foot thickness.

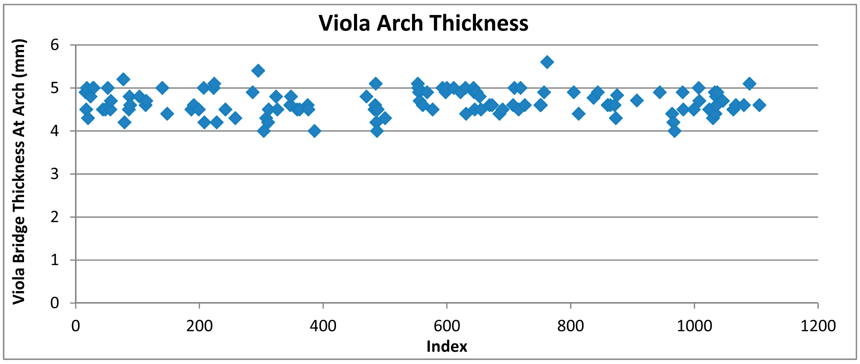


Figure 114. Raw data bridge arch thickness for viola dataset.

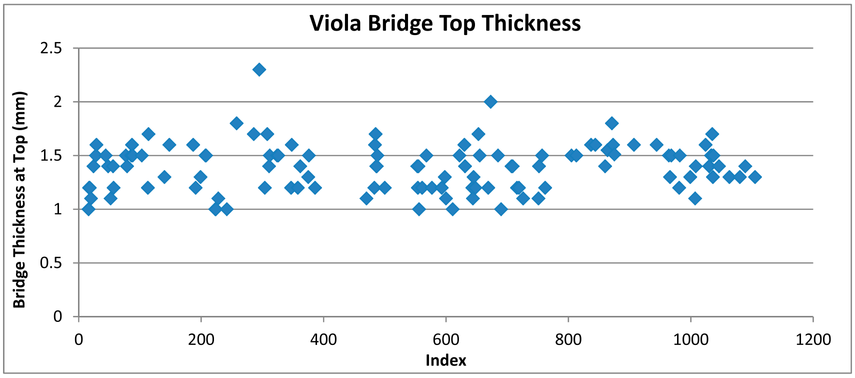


Figure 115. Raw data bridge top thickness for viola dataset.

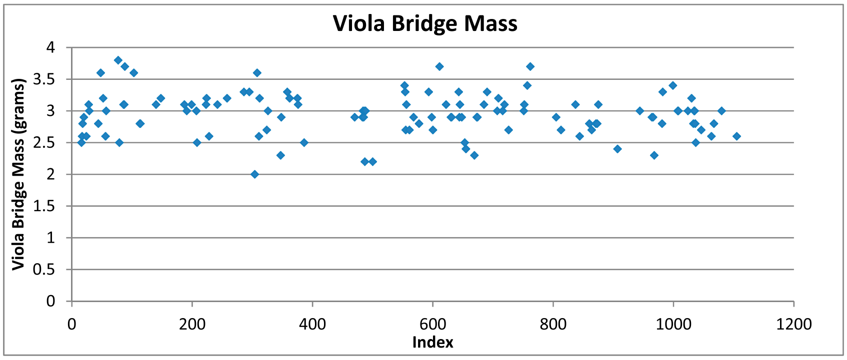


Figure 116. Raw data bridge mass violas.

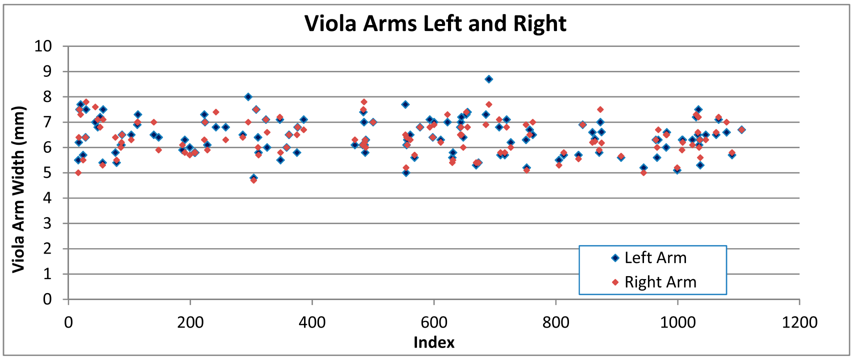


Figure 117. Raw data viola bridge arm thickness.

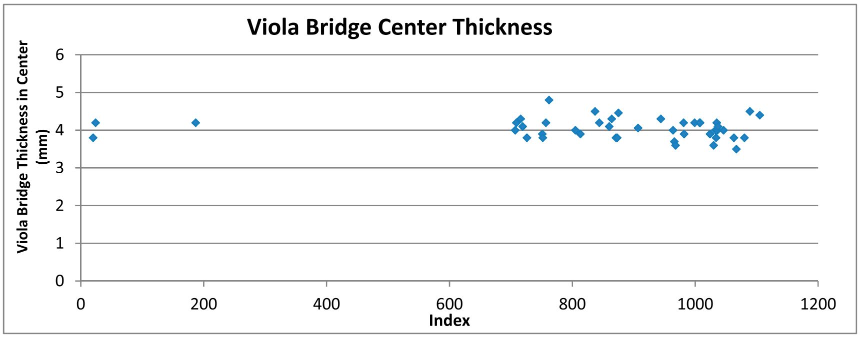


Figure 118. Raw data viola bridge center thickness.

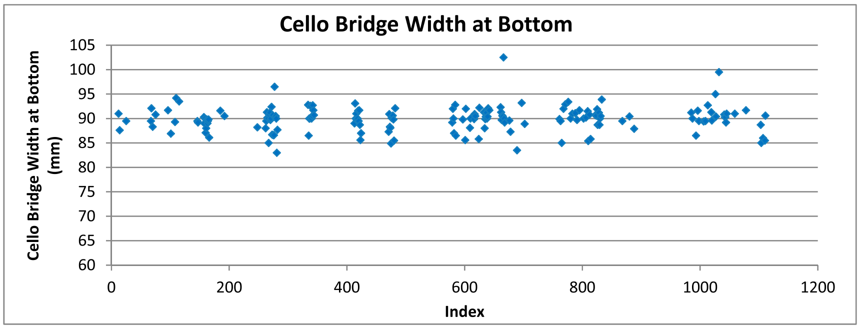


Figure 119. Raw data cello bridge width at bottom.

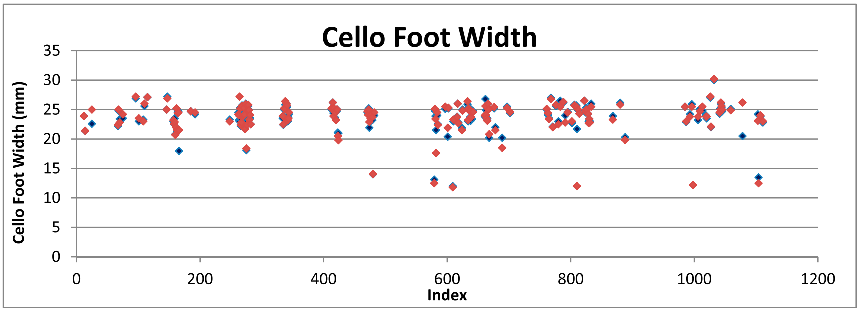


Figure 120. Raw data cello foot width.

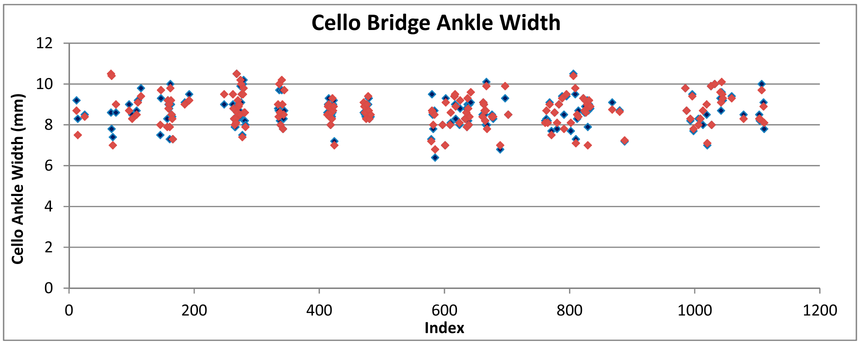


Figure 121. Raw data cello ankle width.

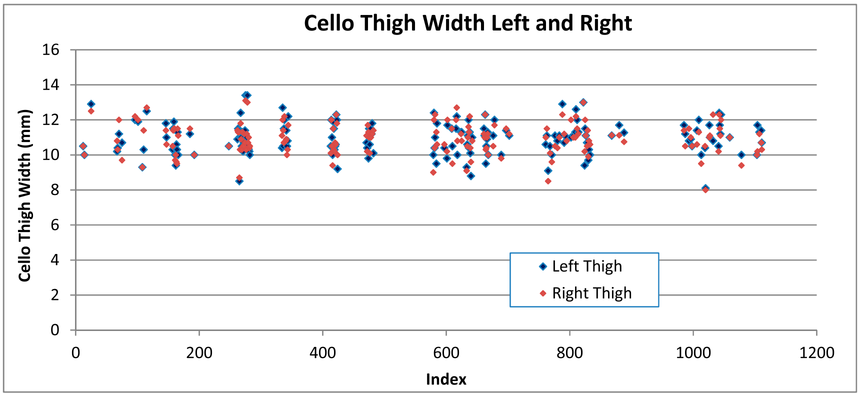


Figure 122. Raw data cello thigh width left and right.

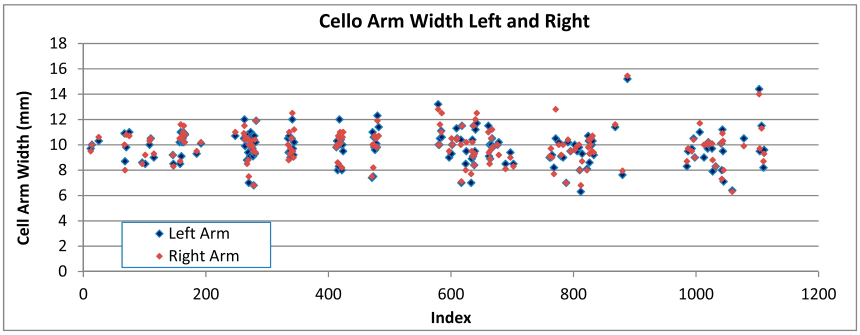


Figure 123. Raw data cello arm widths left and right.

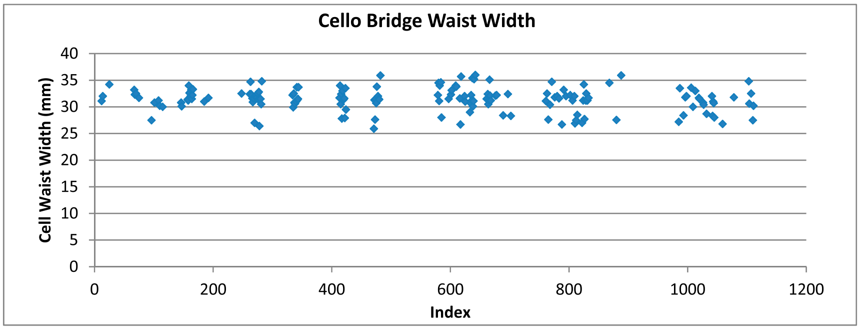


Figure 124. Raw data cello waist width.

Cello Vertical Profile

For the vertical profile, a different technique was used. First, the data set was split into Belgian data and French data because the geometries are quite different. Like the violin data, it was found that the data were not easy to process with multiple profile traces. The data set was sorted based on bridge mass and put into a series of bins. The data from six instruments were then averaged for the French bridges and four instruments for the Belgian bridges. Not all bins and associated averages are presented. The French bridge profiles are shown in Fig. 91.

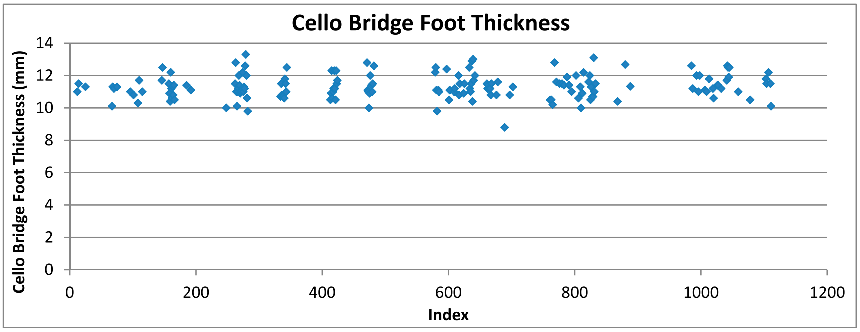


Figure 125. Raw data cello foot thickness.

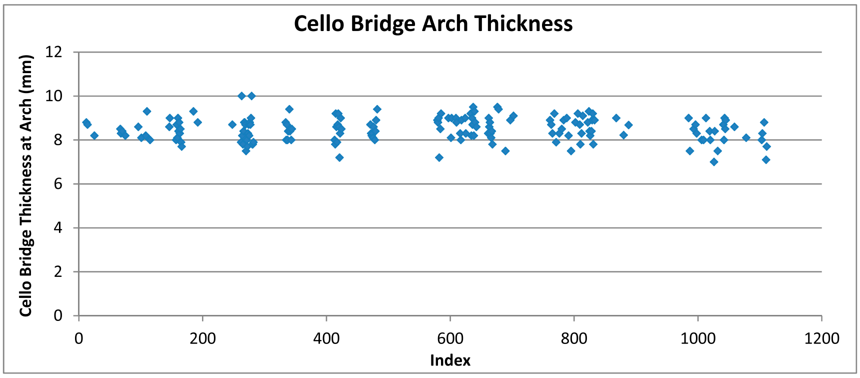


Figure 126. Raw data bridge arch thickness.

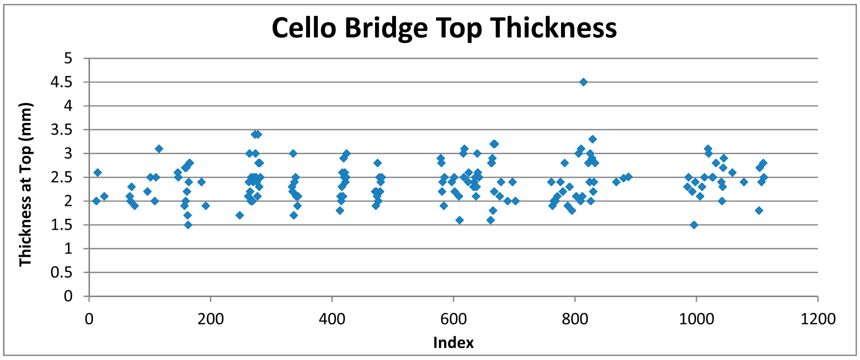


Figure 127. Raw data bridge top thickness.

With the longer legs in proportion to a violin or viola bridge, the profile is more gradual, or more like an A. Mass is clearly affected by thickness. Interestingly, the middle data show that the feet were preferentially thinned over the center. The heavier bridges in this example also have a thicker top. The Belgian data are shown in Fig. 92.

The effect of the longer leg proportion can be clearly seen. The Belgian bridges show more of the steeper profile at the top that was typically seen in violin and viola bridges. In these data, a thickness and mass correlation is seen. Note that, like the violin and viola data, assumed distances above the feet were used.

This is a natural transition into lateral thinning. An indicator has been used for this. The indicator has been changed slightly from the violin data, but is the same as for the viola. In this case, the data have been plotted as arm thickness divided by center thickness, instead of center thickness divided by arm thickness (Fig. 93). It is the same underlying data, just a slightly different presentation.

BRIDGE TERMINOLOGY

Basic bridge terminology is shown in Fig. 94 (Table 1).

DATA SCREENING

The data screening is correlated to instrument number, which is the same as on the publically available database. Baroque bridges were removed from the statistical analysis.

The cello bridge data contained a number of fractional 3/4-, 1/2-, and 1/4-sized data. These were removed as well. There are not quite enough data on the fractional sized cellos to generate good statistics. However, these may become available in the future. The names have been left in the stamps column; however, the data were removed. Notes have been added to the occasional entry under stamps.

The data screening plots, shown in Figs. 95–127, were used to find anomalous data. The photos were then checked for consistency. Where editing was carried out on values based on pictures, the box has been shaded green and the font turned to dark green as well. There are a few holes where measurements could not be estimated as there is no picture, or the picture does not allow measurement. The latter is generally thickness. They have been marked on the spreadsheet with yellow highlights on the boxes. If and when the data can be verified or adjusted, the data can be simply added to the spreadsheets.

CONCLUSIONS

Hopefully, the statistical analysis is of considerable help to those trying to advance stringed instrument construction and design and to those who are actively cutting bridges and want to know whether their dimensions are within normal ranges. With modern computer analysis, bridge design may see some advances in the coming years. There are now some advertisements for carbon fiber bridges, and this means the design of bridges is being actively revisited.

The considerable work that went into creating the database should also be acknowledged and the generosity of those contributing data.

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