Conchoidal Curves for the Violin maker: A Geometrical Solution to the Long Arch of the Violin

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Abstract

This article explores the possible use of a geometrical system to describe the traditional shape of the long arch of the violin family instruments. It presents a historical context based on period documents on which a hypothesis is established. A solution to the geometrical problem is described and then compared with actual period instruments arching shapes. The results show a good correlation to the samples, which appears to validate the efficiency of the solution.

A GEOMETRICAL SOLUTION TO THE LONG ARCH OF THE VIOLIN

Since the 19th century, violin enthusiasts have studied the masterpieces of the famous Italian violin makers. Comparing their properties and shapes became a common interest for musicians, collectors, violin makers, and dealers. Although a large selection of books and articles were written about the techniques and the models of the Cremonese masters, an aspect of the violin anatomy have always been hard to describe: its arching. Terminology often fails to precisely describe this odd shape, authors refer to it as full, pyramidal, cylindrical, tube-like, and some even compared it to a duck's breast. Contemporary violin makers now benefit of a better understanding of the origin of its shape and function, but still speculate on some aspects of the techniques that could have been used. The shape of the long arch is one of them.

Quentin Playfair [15,16] already demonstrated how the Italian violin makers might have sculpted their arching by using cycloid curves.[1] In his article, he compares numerous examples of violin arches with the cycloid curve. The close correspondence has now convinced many that old Italian makers could have known and used these curves. This geometrical solution for the arching is now acknowledged by many, at least as an archetypal shape. Unfortunately, cycloid curves aren't suitable for the long arch of the violin (Fig. 1A). The cycloid has a distinctive sweep that doesn't fit the long arch of any well-made violin, viola, or cello. Since it is necessary to define the shape of the long arch before considering the design of the cross arches, we are still far from a satisfying solution to a comprehensive system.

The catenary curve is another clever option known to have a close connection with the arching of the violin. Torbjörn Zethelius [17] explored the idea that the use of a small chain[1] as a template could guide the carving of the arching. Although his solution is a very clever way to manipulate the catenary as the plates are carved, this method appears to lack the straightforwardness, which is the trademark of the Cremonese masters. Using a real chain as a template is not always convenient. Even if the catenary works well for the long arch of the back, modern makers have yet to find a solution suitable for both the top and the back of the violin (Fig. 1B).

CONCHOID OF NICOMEDES

There is another type of curve that could be considered as a promising candidate to generate long arch shapes. The conchoidal curve (meaning shell-like) was allegedly discovered by Nicomedes

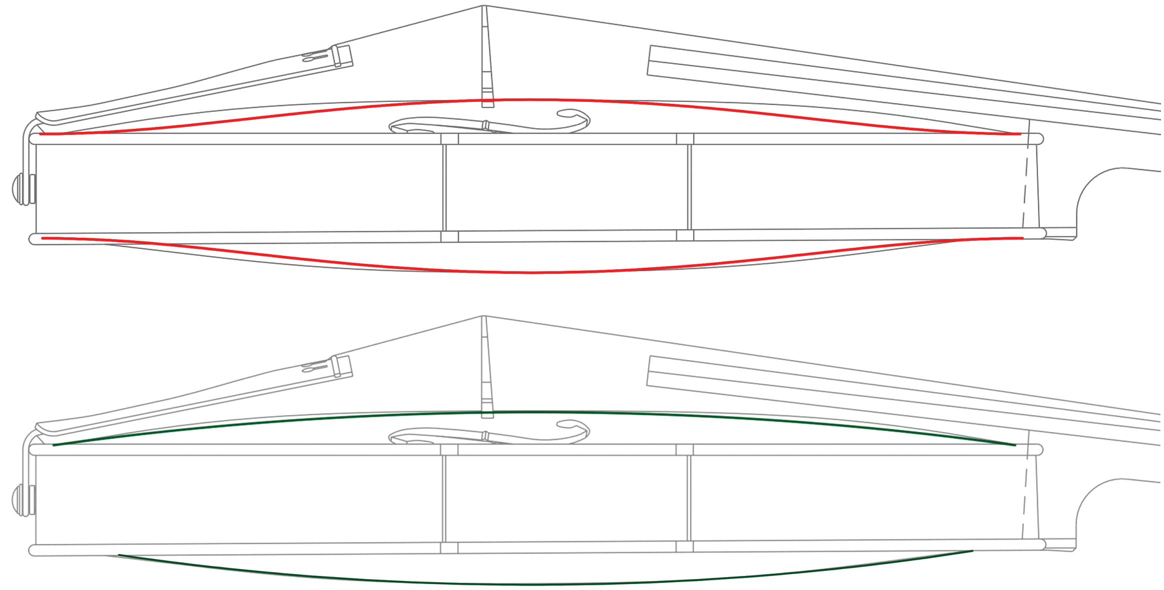


Figure 1. (A) The cycloid is unsuited for the long arch. (B) The catenary can be suitable for the violin back, but not for the top.

of Alexandria in 180 BC. Invented at first as an attempt to solve the trisection of an angle, it led to the study of the asymptotic curves [14]. A device, consisting of a moving ruler guided by a T-square, must be made to properly draft this curve (Fig. 2).

The mechanical device is very simple to make. Two rulers are linked perpendicularly over which a third ruler can slide. Each ruler is slotted to receive dowels that will guide the movement of ruler 1 over the fixed rulers 2 and 3. The dowel Q is fixed on the ruler 1 and slides in the slot of ruler 2. The dowel O is attached to ruler 3, so it is free to slide into the slot of ruler 1. The length of PQ sets the height of the curve while OB changes its shape.

The geometrical construction could be described as a straight line OBN that is rotating center O, and that crosses the line CD. The curve is generated by the movement of a point P (on the rotating line OBN located at a given distance NB from the line CD. The curve is asymptotic: it endlessly gets closer to the line CD while never touching it (Fig. 3).

Finding the appropriate parameters to generate a convincing arching is not as trivial as one could wish. The violin maker will have to adjust by trial and error the dimensions for OB and BN to find good combination. It is, however, possible to produce nice sets of curves similar to acknowledged violin arching shape (Fig. 4).

Although the existence of the conchoidal could have been made accessible to the Italian masters through the work of the Bolognese mathematician Mario Bettini (1582–1657) (Fig. 5), it does not seem to integrate naturally into the proportional measurement and geometrical system we are more familiar with for the violin design at that time.[2]

CONCHOID OF DÜRER

The conchoidal curve, or rather its main idea, might have been already used in the workshops of the 16th century if we consider the book *De Symmetria und Underweysung der Messung* by Albrecht Dürer [9].

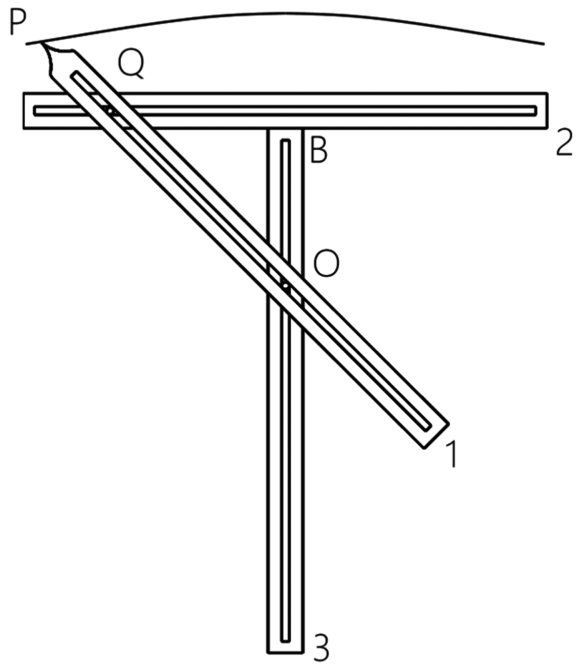


Figure 2. Mechanical device used to draft Nicomedes' conchoid.

Dürer (1471–1528), mostly known for his paintings and engravings, reintroduced the use of geometrical design in the German workshops. During a trip to Italy, he was deeply inspired by the very advanced drawing and design techniques. This apparent technical advance can be explained by the earlier start of the Renaissance movement in this part of Europe. People had already started to look back at the geometrical knowledge of Antiquity. Dürer was highly impressed, and he documented these techniques. His observations were eventually made into a book that mainly covers Euclidian geometry, perspective, proportion, and a collection of decorative and utility shapes, such as spirals (mostly based on Archimedes) and proportions of type letters.

Although his work was of great interest, Dürer didn't bring new knowledge to Italy. He was a witness of its flourishing arts. He didn't consider geometry as an end, as did the Euclidian scholars, but he rather used it like a tool to achieve artistic goals. This might have helped to fill a gap between pure analytical geometry and workmanship. We can guess that his interest in the emerging industry of printing was also helpful for violin makers from Tyrol, France, and northern Italy. The larger production of printed books became more affordable to craftspeople.[3] The fact that it wasn't written in Latin, the language of science, and that it was intended for craftsmen undoubtedly contributed to spread some previously well-kept trade secrets.

There is a chapter in Dürer's book that explains how to make complex and well-balanced curves using line systems very close to the principles elaborated by Nicomedes 17 centuries earlier. The general feel of some of the curves has a striking resemblance with the long arch of the violin (Fig. 6), and the way Dürer uses graduated perpendicular guidelines

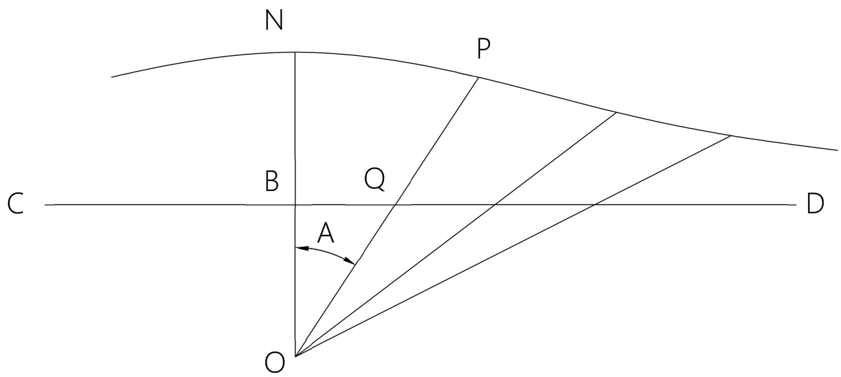


Figure 3. Geometrical description of a conchoidal.

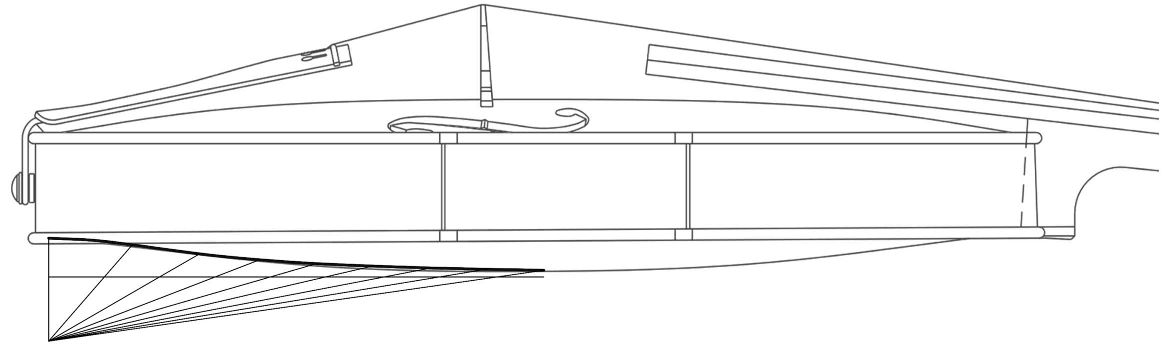


Figure 4. Example of a conchoidal (OB = 30, BN = 18) used to connect the lowest point of the fluting to the apex of the long arch.

APIARIA
VNIVERSÆ PHILOSOPHIÆ MATHEMATICÆ.
IN QVIBVS
PARADOXA.
Et noua pleraque Machinamenta ad vsus eximios traducta,
& facillimis demonstrationibus confirmata,
ILLVSTRISS. ET EXCELLENTISS. D.
MATTHIAE GALASSO.
SAC. ROM. IMPERII COMITI, &c.
A Coll. Tridentino Soc. Iesu dicata.
OPVS
TOMVS PRIMVS.
ACCESSIT AD FINEM SECVNDI TOMI
EVCLIDES APPLICATVS.
AVTORE
MARIO BETTINO BONONIENSI E SOC. IESV.
BONONIÆ.

PROGYM. I.
PROPOSITIO XI.
COROLLARIVM I.

Figure 5. A 1642 edition of Apiaria universae philosophiae mathematicae from Mario Bettini [1] in which he discusses the properties and construction of the conchoidal.

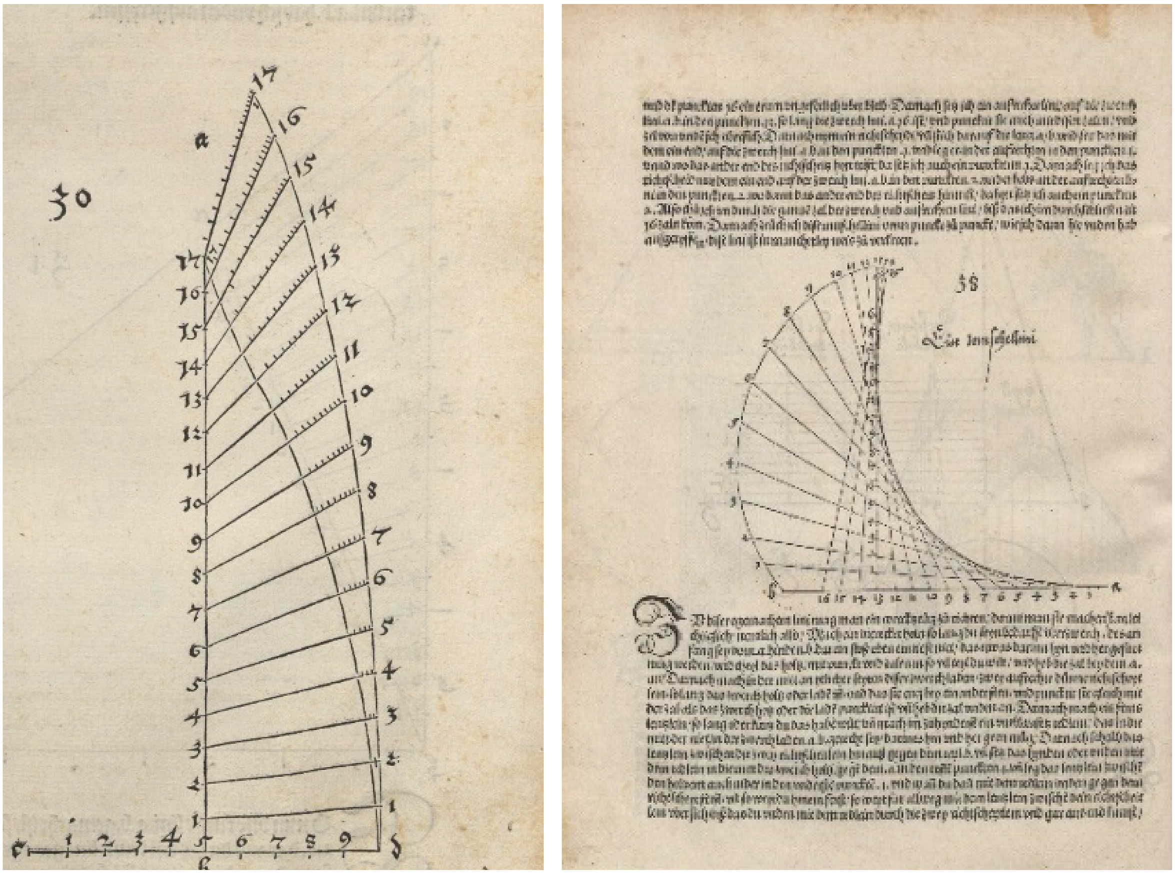


Figure 6. Excerpts from the 1532 edition of De Symmetria und underweysung der Messung from Albrecht Dürer [9].

yields to speculate about violin makers who could have shared the technique.

Dürer's «shell curves» aren't true conchoids but are still often incorrectly referred to as such. The construction method consists of two perpendicular line segments of the same length, both divided by an equal number of parts (Fig. 7). The vertical line CD is positioned on any part of the horizontal line AB (three parts from B for this example). On the prolongation of the line AB, the point E is determined as the starting point of the curve. To trace a second point on the curve, a line segment $A^{I}E^{I}$ (same length as AE) is drawn, so A^{I} is on one part from A and passing through one part above C on the vertical line. The point E^{I} on the other end belongs to the curve.

Composite Solution to Conchoidal Arching Profiles

In its original form, the curve does not appear fit for a violin arching, but by adjusting the parameters and by rearranging the configuration, it is possible to achieve promising results. The following solution presents the advantage of requiring only two main parameters: the length and total height of the plate (the apex of the arching). This approach is also much more consistent with the traditional proportioning system violin makers were already using to design the rest of the violin while still retaining the main characteristics of both Nicomedes' and Dürer's methods.

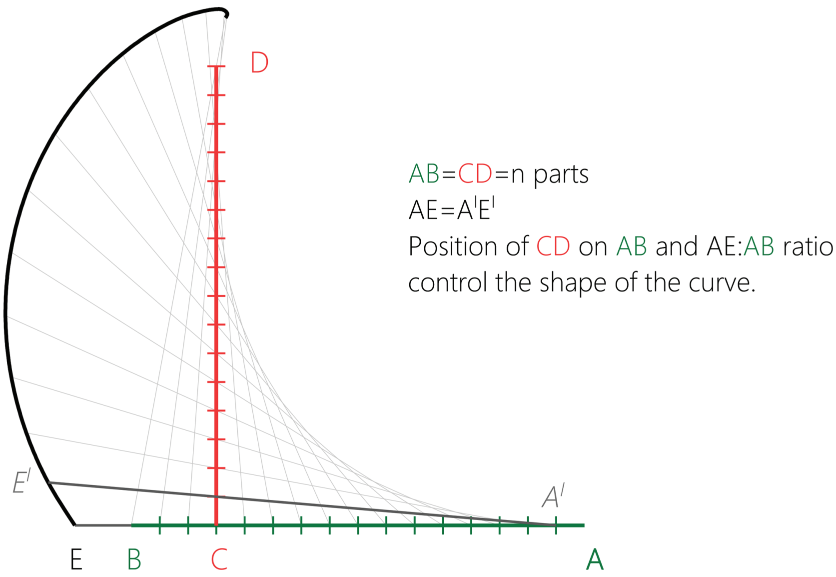


Figure 7. Dürer's construction method for a shell curve (false conchoid).

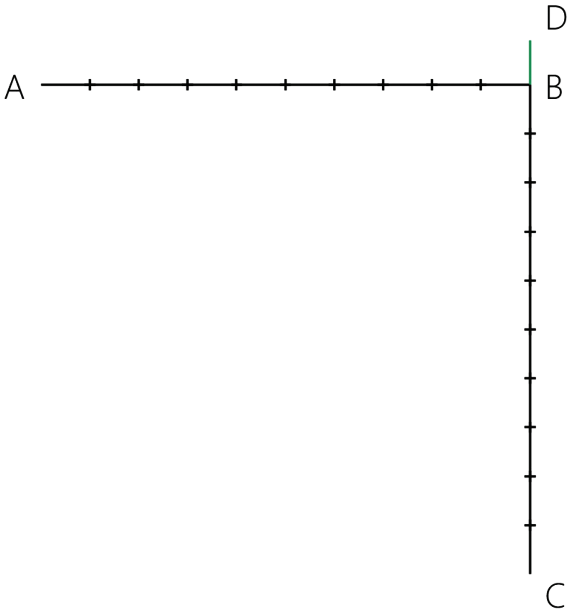
The method starts the same as Dürer's, except that the perpendicular lines are connected to the same end point B. The length of line AB is equal to half the desired length of the violin top or back.[4] Line BC has the same length and is perpendicular to AB. Line BD is equal to the desired total arch height (including the edge). Lines AB and BC are divided into *n* equal parts (Fig. 8A). A diagonal line connects each corresponding part (from bottom to top and right to left as in Fig. 8B). A portion of the diagonal line, equivalent to the height of the arching, is protruding above the line AB. The point on the tip of this line belongs to the conchoidal. Once repeated for each part as in Fig. 8C, the series of points made by the diagonals and apex D can be connected to form a conchoidal curve.

Experimenting with the Method

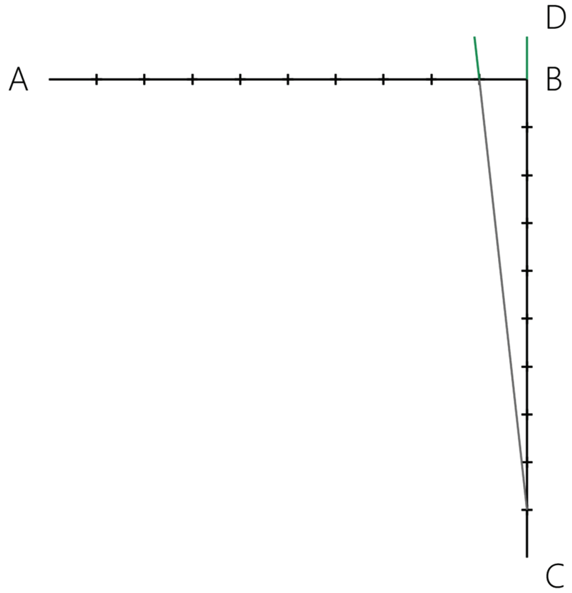
The correlation of this method with the traditional arching shapes was evaluated by comparing the resulting conchoidal curves to actual long arches from random old violins plans. The long arches are based on the plans published in the Strad magazine. Each arching profile have been scanned from the plan, then redrawn in a CAD software to allow accurate measurements between the studied arching and its conchoidal reconstruction. The composite conchoidal method was first applied as described previously, but it soon became obvious that correlation was good for only a few of the samples. The curves were altered to achieve a best fit when needed. These modifications were done by adjusting the length of the line BC while retaining the same number of parts on both AB and BC. The longer BC is, the flatter the top portion of the curve becomes (Table 1). This feature matches nicely the typical arching shape preferred by the Amati family makers.

No manipulation has been made to cope for the possible variation due to eventual cheating in the use of an arching template.[5] In these situations, matching a conchoidal using only the geometrical method described previously becomes very hard to achieve. No attempt to match an off-center highest point have been done, although one could easily draw two individual sets of curves to accurately fit asymmetrical arching profiles. Each studied example is hereafter presented (see Figs. 9 and 10) with the dimensions of AB and BD and the best ratio between AB and BC used to produce the corresponding conchoidal.[6] Only the portion of the conchoidal among the fluting is represented.

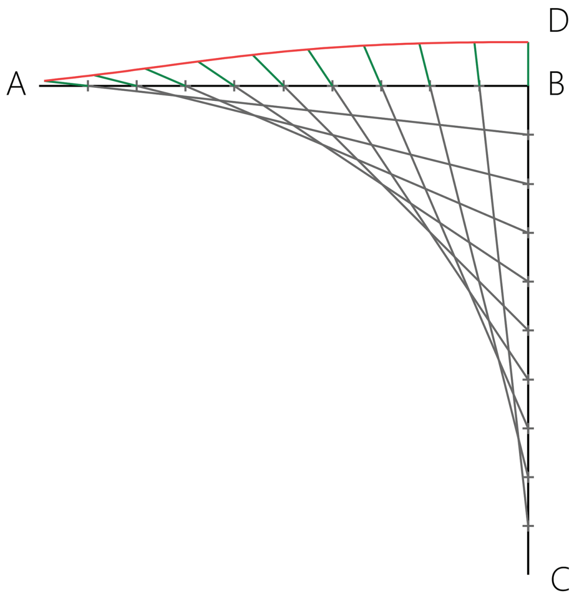
It has been possible to observe a close visual match for most samples. Some distorted arches, probably the result of long-term wood creeping due to string tension pushing down on the violin



8A. AB=BC= half the length of the top or back plate BD= total arching height (including edge) AB and BC are divided in n equal parts.



8B. A diagonal line connects each corresponding part (from bottom to top and right to left) on AB and BC. The protruding end above AB is equal to BD.



8C. The process is repeated for each part of AB and BC until all points are found.

Figure 8. Composite solution to conchoidal arching profiles inspired by Dürer and Nicomedes.

tops, yield a poor correlation with their geometrical reconstruction. A certain amount of distortion might also be imparted to the printing, scanning, and drawing process, although great care has been taken to retain the original proportions of the plans.

Table 1. Measurements and ratios of the samples from Strad posters [2,4,5,6,7,10,11,12,18] used to draw the corresponding arching profiles.

Top Arching	Length (mm)	Arching Height (mm)	Ratio AB:BC
A. Amati 1566	338.4	16.4	1:1
C. G. Testore 1703	355	17	1:Φ
P. Guarneri of Mantoua 1704	352.6	18.3	2:3
A. Stradivari « Titian » 1715	353.3[1]	15.5	4:7
D. Montagnana 1717	354.6	19.9	1:2/Φ
A. Stradivari « Kruze » 1721	354	14.9[1]	2:3
G. B. Guadagnini(viola) 1785	399.75	19[1]	~1:1
Back Arching	**Length (mm)**	**Arching Height (mm)**	**Ratio AB:BC**
A. Amati 1566	341.2	14.1	2:3
G. P. Maggini 1630	368.2	15.2	1:2/Φ
J. Stainer 1650 (viola)	463	20.75	1:2/Φ
C. G. Testore 1703	356	16	1:2/Φ
P. Guarneri of Mantua 1704	354.8	17.8	1:1
A. Stradivari « Titian » 1715	353.3	14.9	1:1
D. Montagnana 1717	355	15.6	1:1
A. Stradivari « Kruze » 1721	356	16[1]	1:1
G. B. Guadagnini (viola) 1785	402	19[1]	~9:10

[1]Estimated dimensions as measured directly on the plan.

ANALYSIS

Fifty-one measurement points have been taken on each curve to compare the *y*-axis values of the original curve to its composite conchoidal reconstruction. The measurement points were evenly spaced along the portion of the curve comprised among (but excluding) the fluting at the edges where a disruption from the main arch becomes apparent.[7] Values for maximum height differences, standard deviation, and correlation coefficient were produced (highest and lowest values highlighted in bold) (Table 2).

The maximum height differences show values varying between 2.1 and 0.6 mm for both top and back archings, with an average of 1 mm. In most cases, the maximum height differences occur at the ends of the curves, where the fluting blends with the convex part of the arching into a smooth transition, or in the bridge portion of the top, where the plates have collapsed under string pressure.

The average standard deviation is about 0.21 and 0.19 mm for both top and back archings respectively. The correlation coefficients proved astonishingly high, with an average of 0.994, the lowest being found in the A. Amati 1566 violin back (0.986). Even if these statistical numbers are good, the differences between the original arching shape and its conchoidal reconstruction can still appear significant to the eye of the trained violin maker.

CONCLUSION

Asserting that the Cremona's golden period masters were using the conchoidal for the long arches of their violins would be rather bold. But even

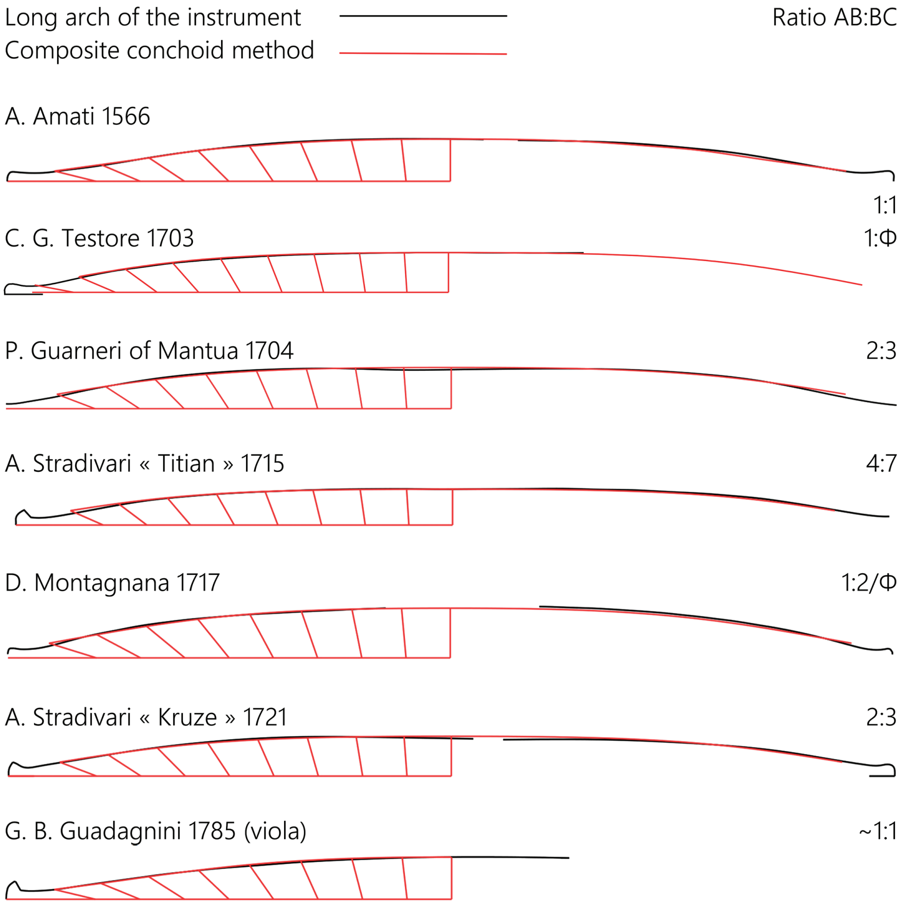


Figure 9. Top arching curves comparison.

if it is impossible to demonstrate the use of this system back then, it remains a very useful method to design arching shapes similar to the well-made examples that are still inspiring contemporary makers. By combining the conchoidal for the long arch and the cycloids for the cross arches, the violin maker now has a more complete solution to help rationalize this part of the instrument.

It is important to note that this process does not allow to create the entire shape of the arching. It only generates the curve among the fluted edges. This should not be a problem if someone is working accordingly to the Cremonese method. The general shape of the arching is roughly carved out first and then thinned to the desired thickness with a little extra wood near the edges. It is only once the purfling is glued in place that the edges are scooped out and blended into the arching. Using only the main portion of the curve template can still be helpful to rough out the arch and to control symmetry.[8]

Skilled and experimented contemporary violin makers, just like the old renowned Italian masters, might not feel the need for such templates to carve a nice arching. Nevertheless, luthiers of the 17th and 18th centuries seldom worked alone. Using templates would certainly have been useful to control and standardize the work of their apprentices. Even though there is no material evidence for the use of such templates, this method can be helpful to less experienced makers or to those aiming at repeatable results.

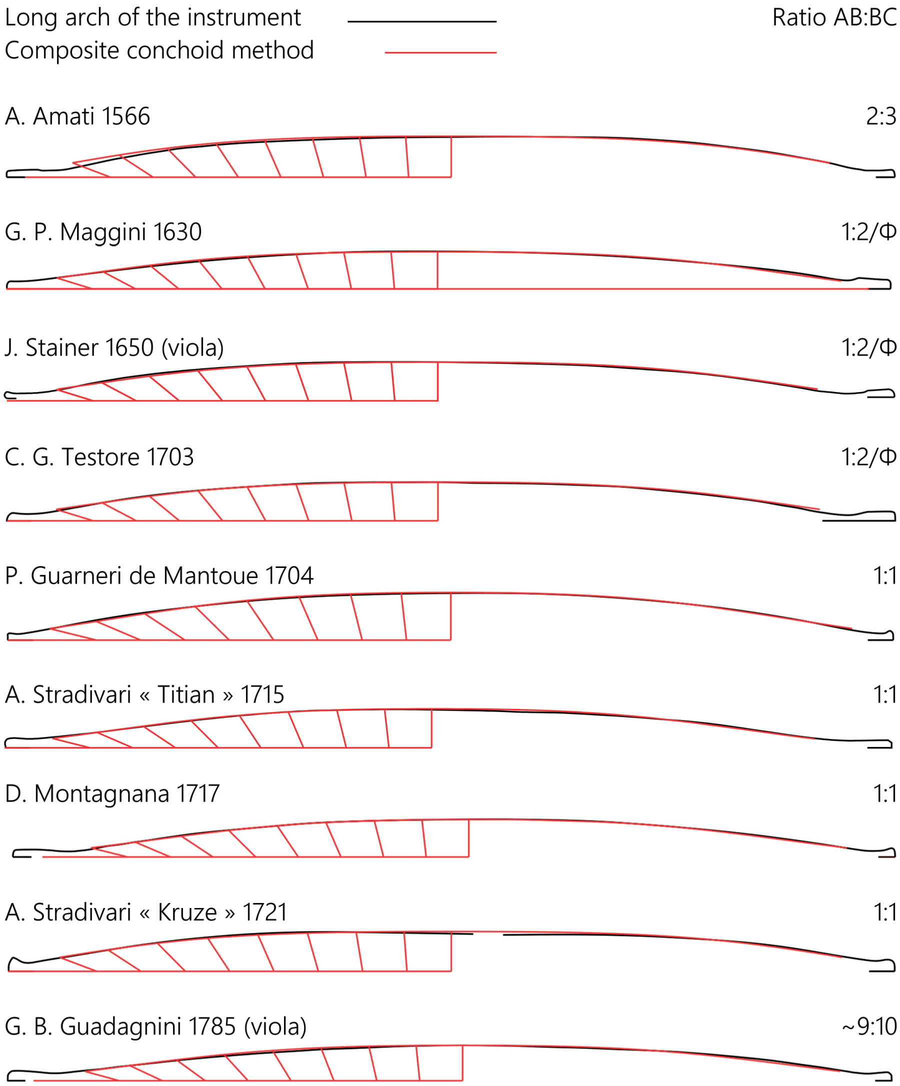


Figure 10. Back arching curves comparison.

NOTES

1. In 1691, Huygens changed its name for catenary, from *catena*, the latin word for chain.
2. See François Denis, book *Traité de lutherie: The violin and the art of measurement* for a thorough study on the subject [3].
3. It is worth reminding that there is no solid evidence to support such a direct influence on the violin design, yet there seems to be a connection between Dürer's type letters and the painted instruments of Andrea Amati. They both use to the same letter types described in Dürer's book. See A. Dipper [8].
4. To simplify the establishment of the value AB when studying actual instruments, half of the total plate length has been used to avoid bias or interpretation for this measurement. Nevertheless, the value of AB can be defined independently from the actual length of the plate to accommodate different opinions about where the arching should start from the plate edges.
5. That is, if the maker tilted or offset a preexisting template to accommodate a different arch pattern.

Table 2. Analysis of the discrepancies between the original arching profiles and their geometrical reconstructions using the composite solution to the conchoidal (highest and lowest values highlighted in **bold**).

Top Archings	Maximum Height Differences (mm)	Standard Deviation	Correlation Coefficients
A. Amati 1566	0.94	0.18	0.997
C. G. Testore 1703	0.69	**0.14**	**0.999**
P. Guarneri of Mantoua 1704	1.24	0.23	0.989
A. Stradivari « Titian » 1715	**1.29**	0.21	0.990
D. Montagnana 1717	1.16	**0.31**	0.993
A. Stradivari « Kruze » 1721	1.20	0.21	**0.989**
G. B. Guadagnini(viola) 1785	**0.66**	0.18	0.998
Average	1.03	0.21	0.993
Back Archings	**Maximum Height Differences (mm)**	**Standard Deviation**	**Correlation Coefficients**
A. Amati 1566	**2.10**	**0.29**	**0.986**
G. P. Maggini 1630	0.82	0.18	0.995
J. Stainer 1650 (viola)	0.98	0.19	0.994
C. G. Testore 1703	1.15	0.16	0.997
P. Guarneri of Mantoua 1704	0.64	0.12	0.999
A. Stradivari « Titian » 1715	0.88	0.19	0.998
D. Montagnana 1717	**0.61**	**0.10**	**0.999**
A. Stradivari « Kruze » 1721	0.84	0.24	0.995
G. B. Guadagnini (viola) 1785	0.97	0.27	0.993
Average	1.00	0.19	0.995

6. Care has been taken to use whole number ratios and golden ratio ($\Phi \sim 1.618 \ldots$) to determine BC as an attempt to include proportionality into the measurements. Any value can work.
7. Although it is hard to rigorously define this point, it usually falls on the eighth diagonal from the apex.
8. Roger Hargrave points out the same observation in his article *The working methods of Guarneri del Gesù and their influence upon his stylistic development, Arching and thicknessing* [13].

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